

ACT National Curriculum Survey® 2012

Mathematics



ACT is an independent, nonprofit organization that provides assessment, research, information, and program management services in the broad areas of education and workforce development. Each year we serve millions of people in high schools, colleges, professional organizations, businesses, and government agencies, nationally and internationally. Though designed to meet a wide array of needs, all ACT programs and services have one guiding purpose—helping people achieve education and workplace success.

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Introduction

Every three to five years, the ACT National Curriculum Survey® asks educators about what they teach (or don't teach) in their courses and how important they feel various topics in their discipline are for students to know to be successful in these courses and in future coursework. The survey also asks educators for their opinions on educational topics of current interest, such as the college readiness of their students or the implementation of improved standards, such as ACT's College Readiness Standards or the Common Core State Standards.

Prior National Curriculum Survey efforts included educators from middle school through the postsecondary level; for the first time, the National Curriculum Survey 2012 also included elementary school teachers. ACT knows that early learning is important for later high school performance—not only do we have the assessment data to prove it, but we now also have survey data about its importance from the very people who teach it.

The Purpose of the Survey

The National Curriculum Survey is a critical step in the process used to build and regularly update a valid suite of ACT assessments that is empirically aligned to college readiness standards. The survey helps to inform the test blueprint for the assessments (see Figure 1). Results from the assessments are used to validate ACT's College Readiness Standards as well as the College Readiness Benchmarks. (The figure represents only this validation cycle, and does not represent how the Standards and Benchmarks were derived.)

Does the test measure Does the test correctly what it purports? predict performance? National Curriculum Content Predictive Survey Validity Validity Standards College Course Test Review Performance Blueprint Test Scores/ Results College College Readiness Readiness Standards Benchmarks Students & HS Postsecondary Tell me what I know Tell me what students know & how that & need to do to be successful leads to success

Figure 1: The Science of ACT Assessments

ACT is committed to validity research. The first type is research into content validity, designed to answer the critical question: Does the test measure what it purports to measure? Essentially, this involves the validation of ACT's College Readiness Standards, which are built on a foundation of years of empirical data and continually validated through the National Curriculum Survey as well as frequent external standards reviews. The second type of research, into predictive validity, is equally important. Using actual course performance, we answer a second critical question: Does the test correctly predict performance? Constant monitoring allows ACT to ensure that the answer to both of the aforementioned questions is yes.

This science behind our assessments—the evidence base and ongoing research—is critical to answering the key question of what matters most in college and career readiness. The National Curriculum Survey represents ACT's commitment to:

- use evidence and research to develop and validate our standards, assessments, and benchmarks;
- maintain a robust research agenda to report on key educational metrics (The Condition of College & Career Readiness, Enrollment Management Trends Report, and The Reality of College Readiness); and
- develop assessments, reports, and interventions that will help individuals navigate their personal path to success along a kindergarten-through-career continuum.

Accordingly, the following principles have shaped and will continue to drive our development agenda:

- 1. Maximize instructional time.
- 2. Establish reasonable testing times.
- 3. Provide transparent connections between ACT's College Readiness Standards and the Common Core State Standards.
- 4. Leverage technology to enhance student engagement, produce more meaningful results, and share results in a timely fashion.
- 5. Increase the emphasis on evidence-centered design, implementing as quickly as possible given technological advances (such as artificial intelligence scoring).
- 6. Include science as a critical content area in our assessment batteries.
- 7. Reflect the reality that there are multiple dimensions of readiness and success (validated by research).

As a not-for-profit educational research organization, we will use these principles to drive the development and continuous improvement of ACT's current and future solutions, as well as the research agenda associated with them, thereby enabling ACT to fulfill its mission of helping all individuals achieve success.

The Survey Results

ACT makes the results of each National Curriculum Survey public in recognition that ACT's data can help educational stakeholders make more informed decisions about college readiness standards and about the alignment of those standards with assessment and curricula. (Survey results for the ACT National Curriculum Survey 2012 are available at http://www.act.org/research-policy/national-curriculum-survey.)

The present report highlights findings from the Mathematics portion of the survey. Participants in this portion included teachers of elementary school mathematics; teachers of middle school mathematics, pre-algebra, algebra, and geometry courses; teachers of high school courses in pre-algebra, algebra, advanced algebra, pre-calculus, calculus, geometry, statistics, and trigonometry; instructors of developmental mathematics at the introductory college level; and instructors of credit-bearing first-year postsecondary courses in the following subjects: calculus; college algebra; college geometry; finite or discrete mathematics; introductory college mathematics/college preparation; pre-calculus; and probability and/or statistics.

The implications of the survey findings are as follows:

- The re-teaching of some earlier topics in middle and high school mathematics courses may take time away from teaching other content necessary to helping ensure that students are ready for college and career in mathematics by high school graduation.
- Students who may be interested in pursuing careers in fields that require
 mathematics likely need to learn additional skills beyond those currently taught in
 K-12 mathematics courses in order to be ready for higher levels of introductory
 postsecondary mathematics coursework.
- Many K-12 mathematics teachers may not yet be ready to teach to college- and career-ready standards, such as those represented by the Common Core State Standards for mathematics.
- The teaching of mathematics-specific reading comprehension strategies in mathematics courses throughout K-12 may need to be increased in order to fuel and reinforce students' abilities to learn mathematics throughout their educational careers.

In the next section of the report, the findings leading to these implications are described in detail. The final section of the report offers recommendations suggested by the findings and implications, while the Appendix contains detailed information about the survey sampling process.

Findings In Mathematics

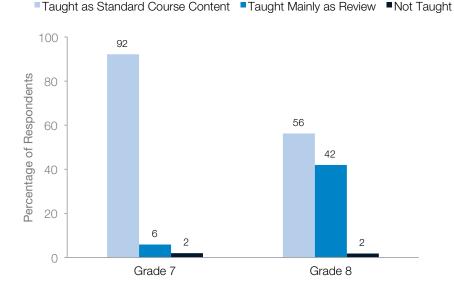
Topic 1: Mathematics and Postsecondary Preparation

Finding 1: Some K-12 mathematics topics that ideally should have been learned before grade 8 are still being taught as standard course content in grade 8 and beyond.

One of the most common critiques of US mathematics education concerns the amount of re-teaching: students being taught many of the same skills in a new course that ideally should have been learned in a previous course or courses. According to critics, if students really understood the topic the first time, then significant amounts of re-teaching could be avoided, and mathematics instruction would be more efficient and effective. As part of the National Curriculum Survey 2012, K–12 mathematics teachers indicated whether topics, if taught in their course, were presented as part of standard course content or mainly as review.

Figure 2 shows the distribution of responses at grades 7 and 8 for one topic of universal interest: Perform addition, subtraction, multiplication, and division on signed rational numbers. This topic is about calculating with integers, fractions, and decimals, and is used throughout life in shopping, tax calculations, and understanding scientific research. It is used for direct computation, problem solving, estimating, and justifying conclusions. In our rank-ordering of responses by instructors of credit-bearing first-year postsecondary mathematics courses, the topic is ranked number 2 in importance as a prerequisite for success in those courses; in some previous years of the survey it has been number 1.

Figure 2: Percentages of Mathematics Teachers at Grades 7 and 8 Reporting Whether or How They Teach the Topic "Perform addition, subtraction, multiplication, and division on signed rational numbers" ¹



¹ Middle school teachers who did not respond to the question were excluded from the analysis.

The figure focuses on these two grades because the Common Core State Standards for mathematics slate regular course instruction of this topic to be completed in grade 7, with fluency increasing as students progress further through school.

The responses show that almost all teachers of grade 7 mathematics courses are covering this topic as a part of regular coursework, while a small percentage considers it review. A smaller percentage does not cover the topic at all, perhaps indicating that students in those classrooms have already learned the topic well enough before grade 7. Overall, the seventh-grade results are consistent with the expectations in the Common Core—if these teachers are in fact finishing up instruction in the topic as standard course content.

After grade 7, the picture is not so ideal. Even before Common Core adoption, state mathematics curricula generally painted a picture of grade 8 in which this topic, if taught at all, is taught as review. This view also persists in the Common Core, presuming as it does that, beginning in eighth grade, teachers would treat this topic only as something that students must use and strengthen as they learn new topics and increase their problem-solving expertise. However, Figure 2 shows that more than half of eighth-grade math teachers report teaching the topic as part of standard course content. In addition, survey results indicate that 45 percent of high school Algebra I teachers and 26 percent of high school Algebra II teachers are also still teaching it as standard course content—suggesting that classroom time in these courses might potentially be taken away from teaching other content necessary to helping ensure that students are ready for college and career in mathematics by high school graduation.

Finding 2: Many of the topics rated as most important by instructors of credit-bearing first-year college mathematics courses are typically covered in high school Algebra I or earlier.

Table 1 shows the 20 topics rated most important as prerequisites by instructors of credit-bearing first-year postsecondary courses. Nine—or 45 percent—of the topics (shaded in blue in the table) are typically covered in grade 7 or earlier, while ten are topics from Algebra I. The one remaining topic (shaded in green in the table) is typically taught in Algebra II.²

²These results are described in terms of a traditional math course sequence, but they apply equally well to an integrated math sequence.

Table 1: Ranking of the 20 Topics Rated Most Important as Prerequisites by Instructors of Credit-Bearing First-Year College Mathematics Courses³

Perform addition, subtraction, multiplication, and division on signed rational numbers Solve linear equations in one variable Solve multistep arithmetic problems Locate points on the number line Perform operations (add, subtract, multiply) on linear expressions Find the slope of a line Find equivalent fractions Perform operations (add, subtract, multiply) on polynomials Locate points in the coordinate plane Write expressions, equations, or inequalities to represent mathematical and real-world settings Evaluate functions at a given value of x Graph linear equations in two variables Order rational numbers Determine the absolute value of rational numbers Manipulate equations and inequalities to highlight a specific unknown Manipulate expressions containing rational exponents Solve linear inequalities in one variable Solve problems using ratios and proportions	Rank	Topic				
2 numbers 3 Solve linear equations in one variable 4 Solve multistep arithmetic problems 5 Locate points on the number line 6 Perform operations (add, subtract, multiply) on linear expressions 7 Find the slope of a line 8 Find equivalent fractions 9 Find and use multiples and factors 10 Perform operations (add, subtract, multiply) on polynomials 11 Locate points in the coordinate plane 12 Write expressions, equations, or inequalities to represent mathematical and real-world settings 13 Evaluate functions at a given value of <i>x</i> 14 Graph linear equations in two variables 15 Order rational numbers 16 Determine the absolute value of rational numbers 17 Manipulate equations and inequalities to highlight a specific unknown 18 Manipulate expressions containing rational exponents 19 Solve linear inequalities in one variable	1	Evaluate algebraic expressions				
4 Solve multistep arithmetic problems 5 Locate points on the number line 6 Perform operations (add, subtract, multiply) on linear expressions 7 Find the slope of a line 8 Find equivalent fractions 9 Find and use multiples and factors 10 Perform operations (add, subtract, multiply) on polynomials 11 Locate points in the coordinate plane 12 Write expressions, equations, or inequalities to represent mathematical and real-world settings 13 Evaluate functions at a given value of <i>x</i> 14 Graph linear equations in two variables 15 Order rational numbers 16 Determine the absolute value of rational numbers 17 Manipulate equations and inequalities to highlight a specific unknown 18 Manipulate expressions containing rational exponents 19 Solve linear inequalities in one variable	2					
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9 Find and use multiples and factors 10 Perform operations (add, subtract, multiply) on polynomials 11 Locate points in the coordinate plane 12 Write expressions, equations, or inequalities to represent mathematical and real-world settings 13 Evaluate functions at a given value of <i>x</i> 14 Graph linear equations in two variables 15 Order rational numbers 16 Determine the absolute value of rational numbers 17 Manipulate equations and inequalities to highlight a specific unknown 18 Manipulate expressions containing rational exponents 19 Solve linear inequalities in one variable	7	Find the slope of a line				
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Manipulate expressions containing rational exponents Solve linear inequalities in one variable	16	Determine the absolute value of rational numbers				
19 Solve linear inequalities in one variable	17	Manipulate equations and inequalities to highlight a specific unknown				
	18	Manipulate expressions containing rational exponents				
20 Solve problems using ratios and proportions	19	Solve linear inequalities in one variable				
	20	Solve problems using ratios and proportions				

This finding suggests that an important contributor to students' college and career readiness is the ability of teachers throughout K–12 to keep strengthening many of the topics students learn in earlier grades, to develop connections, deepen understanding, or add fluency. It's not enough for students to stay at the same level: there is an adage that students really learn the skills from one mathematics course when they take the next course.

At all grade levels, the body of previously learned mathematics remains important as new skills are taught. The survey results show that many topics retain importance in the years after the topic has been covered. For example, as discussed earlier, coverage of the topic "Perform addition, subtraction, multiplication, and division on signed rational numbers" should ideally be completed by the end of seventh grade, and revisited in later years only if necessary. Nevertheless, its importance continues into the more advanced K–12 mathematics courses, even as new topics are introduced in those courses (Figure 3).

Key

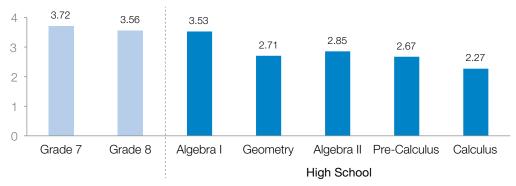
Typically taught in grade 7 or earlier

Typically taught in Algebra I

Typically taught in Algebra II

³ The ranking was determined by sorting the averages of the importance ratings assigned to each topic from highest to lowest, with a ranking of 1 indicating the highest average rating.

Figure 3: Average Importance Ratings Assigned by Teachers of Selected K–12 Mathematics Courses to the Topic "Perform addition, subtraction, multiplication, and division on signed rational numbers"⁴



Finding 3: The topics rated as most important by instructors of first-year college mathematics courses more advanced than College Algebra may indicate what will need to be emphasized in K–12 curricula if the nation is to produce enough graduates to keep up with job requirements in STEM fields.

Various projections, including ACT's, show the number of STEM graduates (science, technology, engineering, and mathematics, often interpreted loosely) as being insufficient to meet the demand for jobs in these fields. Students in these areas are likely to have greater job prospects and to earn a higher salary than they might in many other fields.

Students whose interests, aptitudes, and plans involve STEM coursework typically start college in mathematics courses more advanced than College Algebra. This placement allows the students to take advanced courses earlier in their college career, courses that have one or more mathematics prerequisites. As important as it is to ensure that all high school graduates are prepared for postsecondary courses such as College Algebra, it is no less important to support students interested in STEM by giving them accurate information about the skills they will need to demonstrate in order to place into introductory college mathematics courses beyond College Algebra.

For students to be prepared for first-year college coursework more advanced than College Algebra, such as pre-calculus, calculus, or probability/statistics, the command of certain prerequisite topics becomes especially important. Table 2 shows topics that, with a handful of exceptions, are considered more advanced than the prerequisites typically needed for College Algebra (as expressed by the percentages of postsecondary respondents who rated the importance of the topic as a prerequisite for a given course as 3 or 4, on a scale where 0 means "Not Important" and 4 means "High Importance"). A topic is included in the table if its percentage for pre-calculus, calculus, or probability/statistics is at least 10 points higher than its percentage for College Algebra. (For reference, the College Algebra percentages are also presented.)

⁴ Importance ratings in the survey were labeled as follows: 0=Not Important, 1=Low Importance, 4=High Importance. Teachers who did not respond to the question were excluded from the analysis.

Table 2: "More Advanced" Mathematics Prerequisites, as Determined by Percentages of Instructors of Selected Credit-Bearing First-Year College Courses Who Assigned the Topics Importance Ratings of 3 or 4

Topic	College Algebra	Pre- Calculus	Calculus	Probability/ Statistics
Solve absolute value equations and inequalities	62	76		
Exhibit knowledge of radian measure and the unit circle	7	36	77	
Use the unit circle to find trigonometric values for any angle	7	38	73	
Apply properties of trigonometric functions and their graphs, including amplitude, period, and phase shift	6	32	60	
Apply trigonometric identities	6	32	65	
Use inverse trigonometric functions to solve right-triangle problems	6	29	60	
Find the area of an acute or obtuse triangle using trigonometric functions	6	20	33	
Use the law of sines and law of cosines	5	24	37	
Manipulate radical expressions	72	84	82	
Solve radical equations	46	62	71	
Solve rational equations	57	74	78	
Solve quadratic inequalities	30	49	49	
Determine the number and types of solutions of a quadratic using the discriminant	35	48	48	
Determine solutions of polynomial and rational equations	44	63	75	
Solve problems using equations of parabolas and circles	24	41	55	
Solve problems using equations of ellipses and hyperbolas	7	15	30	
Implement remainder and factor theorems for polynomials	19	30	37	
Find the domain and range of a function	59	76	79	
Evaluate a composite function at a given value of x	40	56	83	
View a function as a composite of two functions	24	42	72	
Write a composite function to represent mathematical and real-world relationships	19	27	58	
Solve problems using the relationship between logarithmic and exponential functions	25	40	69	
Find the inverse of a function	29	44	61	
Write the equation of an ellipse or a hyperbola	10	18	35	
Find magnitude, direction, and components of vectors	6		17	
Perform operations (add, subtract, multiply by a scalar) on vectors	7		16	
Solve problems involving vectors	4		14	
Find the limit of an expression	8		52	
Write arithmetic and geometric sequences both recursively and with an explicit formula	9		26	
Solve problems using parametric equations	4		22	
Use a statistical package	5			29
Understand the role of randomization in surveys, experiments, and observational studies	8			55
Exhibit knowledge of correlation, variance, and standard deviation of data	9			46
Fit a normal distribution to a data set	5			35
Fit a function to a data set (linear, quadratic, exponential)	13			39
Determine the number of elements in the sample space using combinations and permutations	6			37
Recognize and explain the concepts of conditional probability	4			41
Construct and interpret two-way frequency tables	3			42
Find union and intersection of sets	26			53

This list of topics can serve as one possible guide to the kinds of skills that students will need to learn—and high school courses will need to teach—if the nation is to meet the challenge of producing more high school graduates able to pursue STEM majors in college and, ultimately, STEM careers.⁵

Topic 2: Mathematics Instruction and the Common Core State Standards

Finding 4: While educators acknowledge the importance of several aspects of the Common Core State Standards for mathematics, they are perhaps not yet as familiar with the standards as they may ultimately need to be.

The Common Core State Standards Initiative, a state-led effort to develop collegeand career- ready standards for English language arts and mathematics across the K–12 grades, is a landmark development in US public education. The standards resulting from the initiative align the two subject areas with a uniformly higher standard than has been the norm in the past: readiness for college and career by high school graduation.

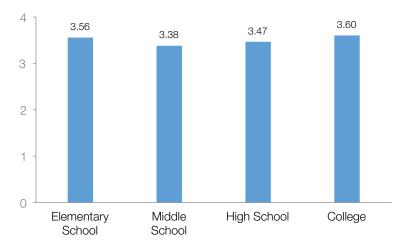
In addition to specific grade-level standards from kindergarten through high school, the Common Core State Standards for mathematics includes a section called the Standards for Mathematical Practice. This section sets forth eight practices that are important for students at all grade levels, because these practices rest on important "processes and proficiencies" of longstanding importance in mathematics education (Common Core State Standards Initiative, 2012). Organizations such as the National Council of Supervisors of Mathematics advise teachers to start their transition to the Common Core by focusing some attention on teaching the Mathematical Practices, not only because the organization considers them to be important, but also as a way of starting Common Core implementation without compromising preparation for state accountability measures that may be based on previous state standards.

The ACT National Curriculum Survey 2012 found that, taken as a whole, the Mathematical Practices were judged to be important by teachers at all educational levels.

The most important of the eight Mathematical Practices is arguably the first: "Make sense of problems and persevere in solving them." Figure 4 shows the importance assigned to Practice 1 by teachers of K–12 mathematics in the courses they teach from grade 3 through high school, and by instructors of first-year credit-bearing college mathematics courses as a prerequisite for success in those courses.

⁵ The Common Core State Standards for mathematics has a slightly different selection of advanced topics, marked in the standards document with a plus sign: "Standards marked with a (+) . . . can form the starting point for fourth year courses in Precalculus and in Probability and Statistics. Other fourth year courses, for example Calculus, Modeling, or Discrete Mathematics are possible" (Common Core State Standards Initiative, n.d., p. 147). Whatever list is chosen, it is possible that if a substantial number of these skills are taught in high school, the content of introductory college mathematics courses geared to STEM can eventually be adjusted to begin at a slightly more advanced level.

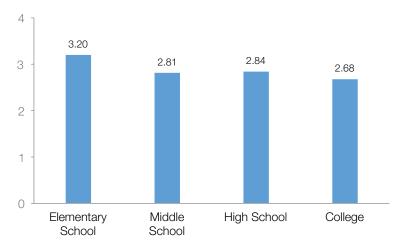
Figure 4: Average Importance Ratings for Common Core State Standards Mathematical Practice 1, "Make sense of problems and persevere in solving them"



This first practice seems to become slightly less important in middle school, and then increases in importance as course content grows more complex. Survey results also show that this practice is taught as standard course content (as opposed to taught mainly as review or not taught at all) by clear majorities of teachers from grade 3 through high school.

For Mathematical Practice 6, "Attend to precision," the pattern is different (Figure 5):

Figure 5: Average Importance Ratings for Common Core State Standards Mathematical Practice 6, "Attend to precision"⁷



As the figure shows, the importance of this practice varies more across educational levels than does the importance of Practice 1. Attending to precision appears to be seen as most important in elementary school and least important in college

⁶ Educators who did not respond to the question were excluded from the analysis. Grade 6 mathematics, high school trigonometry, and high school statistics were omitted due to small sample sizes.

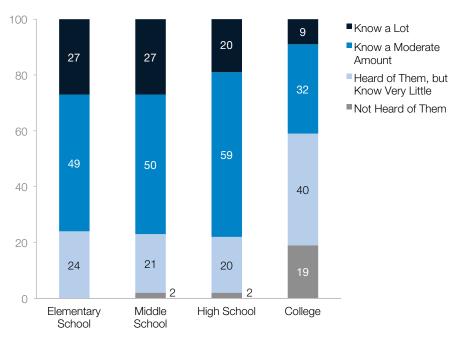
⁷ Educators who did not respond to the question were excluded from the analysis. Grade 6 mathematics, high school trigonometry, and high school statistics were omitted due to small sample sizes.

(although, when broken down by course, it is rated least important overall in high school Algebra I and College Algebra). We see no clear explanation for this pattern. Practice 6 is taught as standard course content in most courses from grade 3 through high school, with the exception of high school pre-calculus.

The other Mathematical Practices show still other patterns. For example, as one might expect, Practice 3, "Construct viable arguments and critique the reasoning of others," is considered especially important in high school geometry. But other patterns in responses remain unexplained.

The lack of clear patterns in many of the responses may reflect the educators' overall lack of in-depth knowledge about the Common Core State Standards for mathematics. Fewer than one-fourth of the K–12 mathematics teachers (24 percent across the three levels), and fewer than one-tenth of the instructors of credit-bearing first-year college mathematics courses (who are not directly affected by the Common Core), reported knowing "a lot" about the standards (Figure 6).

Figure 6: Percentages of Mathematics Educators Reporting Various Degrees of Familiarity with the Common Core State Standards for Mathematics⁸



However, as seen in the figure, taking into account the K-12 teachers who reported knowing a moderate amount about the standards improves the picture to some extent, suggesting that close to 80 percent of these teachers are at least fairly conversant with the content of the standards. In any event, the K-12 teachers who responded to ACT's open-ended survey question about professional development

⁸ Educators who did not respond to the question were excluded from the analysis. Due to rounding, the high school percentages do not sum to 100.

related to the Common Core ("What kinds of professional development do you need and would you find most beneficial for the Common Core State Standards initiative to work?") overwhelmingly expressed the need for training, another sign that these teachers are not yet fully confident in their ability to understand and teach the skills contained in the standards to their students.

Topic 3: Mathematics and Reading

Reading in mathematics might well be considered a genre by reading specialists. There are words that take on technical meanings different from standard English. There are communications conventions, including the use of symbols. Understanding problems often involves identifying quantitative information and relationships among that information. Knowledge of standard mathematical models comes into play. Interpreting graphs, tables, and other representations, and being familiar with typical patterns found in these representations, are unique skills that students must learn. Understanding expressions and attaching meaning to their components is key, and is difficult for many students.

This kind of reading is important for college and career readiness. Not only will students be expected to read and digest quantitative information and argument in college, but students who are able to learn independently have a tool for learning more mathematics throughout their lives. In the past, mathematics standards seldom addressed the need for reading in mathematics. The Common Core State Standards integrate reading across disciplines within the English language arts standards, and make clear that it is the responsibility of teachers in all courses—mathematics no less than any other—to help teach reading. The National Curriculum Survey 2012 investigated several aspects of the interaction between reading comprehension and the expectations of mathematics coursework, both in K–12 and at the college level.

Finding 5: Within K–12, the amount of time spent teaching strategies for reading mathematics course material, and the amount of assigned reading that teachers expect students to read and understand independently, appear to decrease with educational level.

The survey asked how much time mathematics teachers at various educational levels spend teaching students how to read course materials. The results are shown in Figure 7.

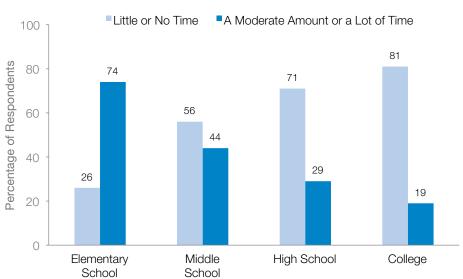


Figure 7: Percentages of Educators Reporting How Much Time They Spend Teaching Students Strategies on How to Read Course Materials⁹

The amount of time spent teaching reading strategies decreases steadily as educational level increases, with elementary school teachers substantially more likely to spend a moderate amount or a lot of time than teachers in the higher levels. One might well expect this: as students improve their reading comprehension skills, they are likely to become better able to learn mathematics by means of the reading strategies they already possess. But given that most elementary school teachers teach a variety of subjects (including reading itself), it is also possible that the time they reported spending may not necessarily be specific to mathematics, but rather to strategies applicable across the curriculum, including mathematics.

Practice, such as the kind offered by engaging with course materials on their own, can help students improve their ability to comprehend mathematics materials. How much of this kind of practice are students likely to get in mathematics courses? Figure 8 summarizes educators' responses to the question of how much of the assigned reading in their courses they expect students to read and understand independently, with little or no instructional scaffolding or support.

⁹ Educators who did not respond to the question were excluded from the analysis.

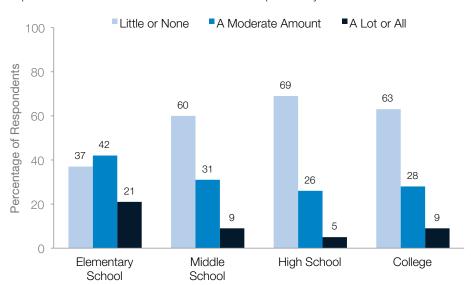


Figure 8: Percentages of Educators Reporting How Much Assigned Reading They Expect Students to Read and Understand Independently¹⁰

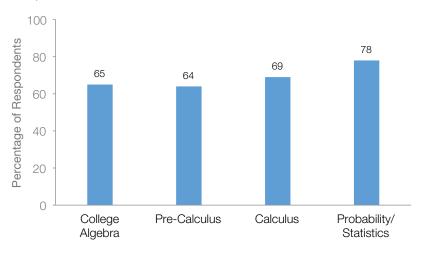
The figure shows that high school mathematics courses tend to require less independent reading than do mathematics courses at other levels, and that elementary school tends to require it the most. (Again, the elementary-school results may refer to more than just mathematics-specific reading.)

Finding 6: Clear majorities of postsecondary mathematics instructors believe that at least half of their students possess an appropriate degree of reading comprehension.

After reviewing some of the instructional practices in K–12 related to reading and reading comprehension, it may be instructive to see how some of the results of these practices might manifest themselves when students get to college. As part of the National Curriculum Survey 2012, instructors of credit-bearing first-year college mathematics courses shared their observations of how well prepared their entering students are for the reading expectations of the course they teach. Figure 9 shows results for four categories of postsecondary mathematics courses covered in this section of the survey: College Algebra, pre-calculus, calculus, and probability/ statistics.

¹⁰ Educators who did not respond to the question were excluded from the analysis.

Figure 9: Percentages of Credit-Bearing First-Year Mathematics Courses Reporting that about Half or More of Their Students Have an Appropriate Level of Reading Comprehension¹¹



Because calculus courses and, in some cases, probability and/or statistics courses are more advanced than College Algebra and pre-calculus courses, it is reasonable to expect that the necessary level of reading comprehension in the two former categories would be somewhat higher than that in the two latter categories. It is therefore encouraging to note that higher percentages of students in calculus and probability/statistics appear to have appropriate levels of reading comprehension than do students in algebra and pre-calculus. This is consistent with students improving their reading comprehension in mathematics as they advance from course to course. However, according to the survey results, students in college mathematics courses have a wide range of reading comprehension in mathematics, and many college instructors—more than one-third, for example, of the respondents in college algebra and pre-calculus—find that, in general, their students have reading skills insufficient to meet the requirements of those courses.

¹¹ Instructors who did not respond to the question were excluded from the analysis.

Recommendations

In light of the preceding results from the National Curriculum Survey 2012 in mathematics, ACT offers the following recommendations to help in the pursuit of college and career readiness for all students:

 To promote teaching of rigorous K-12 courses and good communication about expectations, and to cut down on the need to re-teach topics from year to year, there should be a common vision of what is regular course content for a course, including the depth required.

Given that mathematics topics in K–12 continue to be taught as regular course content long after they should have been learned, guidelines for efficient learning progressions from grade to grade and course to course should be established. While it is certainly expected that teachers will work with students who need help getting up to speed on particular topics, an abundance of re-teaching is a potential detriment to overall course rigor, particularly in high school. ACT knows that teachers are working to strengthen understanding and fluency, but the whole system needs a reboot so that, when topics need to be covered after they have been taught, they are covered almost entirely as review, without sacrificing time spent on the new topics that must be learned in successive courses if students are to stay on target for college and career readiness in mathematics.

Another reason that establishing learning progressions is important is that, while the Common Core State Standards for mathematics show development of many mathematical competencies in small stages across grades, forming a coherent picture of how students can build complex knowledge and reasoning structures, not everything in the Common Core State Standards for mathematics is staged across grades, the Mathematical Practices in particular. Given the importance of the Mathematical Practices, mapping their expected progression across grades should be a high priority, putting them at a level equivalent to other parts of the Common Core.

 Students interested in science and mathematics, many of whom might choose to major and work in STEM fields after high school, need opportunities to learn the additional prerequisite skills associated with advanced introductory-level college mathematics courses.

While preparing all high school graduates for typical introductory postsecondary mathematics courses is a worthy and critical goal, it may not be enough to increase the supply of workers needed for jobs in the fields of science, technology, engineering, and mathematics. To the extent possible, K–12 mathematics courses should offer students opportunities to learn STEM-related skills, potentially to encourage more interest in these fields as well as to better enable interested students to pursue STEM-related coursework in college.

These skills also need to be assessed in K–12, so that the students can know whether they are ready for more advanced levels of introductory college mathematics coursework. By having a STEM component to state assessment, schools will be able to identify and encourage students who show appropriate achievement, with the likely result of increasing the number of STEM majors going to college.

 Given the large number of states adopting the Common Core State Standards, schools need an effective and efficient way to provide professional development support to K-12 teachers while keeping budgets in mind.

As of this writing, 45 states and the District of Columbia have adopted the Common Core State Standards for mathematics. There is a great need for professional development about implementing the standards. While the building blocks of the Common Core are not new, the way the pieces are put together, and the philosophy, represent a change. Many teachers are well prepared, and many others can be with appropriate support. One way of providing this support efficiently is for states and schools of education to band together to find and/ or create high-quality professional development, and to collect data about effectiveness in order to make improvements year by year.

4. K–12 mathematics teachers beyond elementary school need support in teaching mathematics-specific reading strategies.

The survey findings about the percentages of K–12 mathematics teachers who teach students strategies for reading and understanding course materials suggest that more can be done in this area. Students who are fluent in the specific skills for comprehending mathematics materials are better able to learn more mathematics throughout their careers.

Because students weak in reading comprehension in mathematics may also be weak in general reading comprehension, a school-based support system that involves English language arts teachers, as well as teachers from other disciplines, may best address some of these students' needs. A school-based system is also a way to share reading expertise across the school, as well as to make the point that reading across the curriculum is important.

Appendix: Description of Survey Sample and Process

The ACT National Curriculum Survey is a one-of-a-kind nationwide survey of educational practices and expectations conducted by ACT approximately every three years. ACT surveys thousands of teachers and college instructors in English/writing, mathematics, reading, and science for the purpose of determining what skills and knowledge are currently being taught at each grade level—and which are considered essential for college readiness. The survey also asks educators for their opinions on educational topics of current interest.

For the ACT National Curriculum Survey 2012, we sent surveys by postal mail and e-mail to a nationally representative sample of elementary school, middle school, high school, and college instructors who teach courses in English/writing, mathematics, reading (including English language arts and social studies), and science (including Biology, Chemistry, Physics, and Earth/Space Science) in public and private institutions across the United States. We also included a sample of instructors of developmental (i.e., remedial) college courses in English/writing, mathematics, and reading. We included these instructors because they should be uniquely qualified to identify the critical skills and knowledge that high school graduates are typically missing and the set of knowledge and skills that, when emphasized, result in student readiness for success in entry-level college courses. Table 3 gives the numbers of survey participants at each educational level.

Table 3: ACT National Curriculum Survey 2012 Participants

Educational Level	Number of Participants
Elementary school	1,052
Middle school	1,806
High school	2,943
College developmental	540
College	3,596
TOTAL	9,937

The numbers of participants listed in Table 3 compare favorably to those from past surveys. Excluding elementary school teachers, who are new to the survey, the total number of participants in 2012 is 16 percent higher than the number who participated in 2009, and 35 percent higher than the number who participated in 2005–06.

ACT uses the results from the main body of the ACT National Curriculum Survey to guide the test development of ACT's college readiness assessments. ACT conducts this portion of the survey to ensure that its assessments are measuring the current knowledge and skills that instructors of credit-bearing first-year college courses identify as important for success in each content area. As in past years, the results of this section affirm that the knowledge and skills that are important for readiness

and success in college and in workforce training, and the relative emphasis accorded to each, are reflected in the content of ACT Explore®, ACT Plan®, and the ACT® college readiness assessment.

All participants surveyed were asked to perform two primary tasks with respect to course content. First, they were asked to rate discrete content knowledge and skills with respect to how important each is to student success in the content area. (Specifically, high school teachers and college developmental instructors were asked to rate the importance of each content or skill in a given class they teach, while instructors of credit-bearing college courses were asked to rate the importance of each content or skill as a prerequisite to success in a given class they teach.) Second, they were asked to rank groups of content and skills, known as strands, with respect to their relative importance to student readiness for college.

We also asked the K–12 teachers to indicate whether or not they teach a particular content or skill and, if so, whether they teach it as a standard part of their course or as part of a review of material that should have been learned earlier. Finally, we asked all educators a number of questions about, e.g., the amount and type of reading and writing they assign; the textbooks they use; their awareness of the Common Core State Standards and of their state's, school's, or district's alignment efforts across K–13; their students' readiness for particular kinds of coursework; and their students' degrees of reading comprehension, computer literacy, and computer access.

Because some content areas were surveyed in larger numbers than others, the values displayed in educational-level totals were averaged across English/language arts, mathematics, and science. This ensured that, in these results, no one content area would have more influence than another.

References

Common Core State Standards Initiative. (n.d.) Common Core State Standards for mathematics: Appendix A: Designing high school mathematics courses based on the Common Core State Standards. Retrieved September 18, 2013, from http://www.corestandards.org/assets/CCSSI_Mathematics_Appendix_A.pdf

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