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Estimating Conditional Probabilities of Success and Other Course Placement Validity Statistics Under Soft Truncation

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Abstract

ACT's Course Placement Service uses logistic regression to model the relationships between outcomes in standard college courses and placement test scores. The logistic regression results are used together with empirical test score data to obtain estimates of validity statistics (e.g., proportion of correct placement decisions), given particular hypothetical cutoff scores. Prior research has shown that the accuracy of these statistics is affected by *hard truncation,* a condition in which no standard course outcome data are available for students below a certain cutoff score. Hard truncation is uncommon in actual placement systems, however; standard course outcome data typically are available for some students below the cutoff score (*soft truncation*). A soft truncation condition of 40%, for example, is one in which 40% of the students below the cutoff score do not have standard course outcome data. The results of this study show that reasonably accurate estimates of validity statistics can be obtained even when 40%, and in some cases 60% or 80%, soft truncation occurs. Moreover, the slope of the logistic regression curve and the skewness of the test score marginal distribution have little to do with the relative accuracy of validity statistics unless soft truncation exceeds 40%.

Acknowledgments

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- Chuan-Ju Lin, for helping with the somewhat tedious task of generating 12,000 truncation samples; and
- Julie Noble and Richard Sawyer, for offering helpful suggestions for the design of the study.

Estimating Conditional Probabilities of Success and Other Course Placement Validity Statistics Under Soft Truncation

Postsecondary institutions often use standardized test scores for deciding how to place students into college-level courses. Typically, students scoring at or above a particular cutoff score are placed into a standard course, whereas those scoring below the cutoff score are placed into a lower-level course, sometimes referred to as a "remedial" course. It is very important that correct placement decisions be made; otherwise, students may be adversely affected. For example, entering freshmen who are incorrectly placed into a standard course may be unable to complete it with a passing grade because they lack sufficient academic knowledge and skills. This situation could, of course, be disheartening for students.

Placement decisions may also affect an institution. If institutional staff determine that many students need remediation, for example, then staff may attempt to schedule additional remedial courses. If incorrect placement decisions have been made and these students do not, in fact, need remediation, then scheduling efforts will have been needlessly undertaken.

Sawyer (1989, 1996) applied decision theory to evaluating course placement systems. He considered four possible events resulting from a course placement decision:

1. Students who score at or above the cutoff are adequately prepared for the standard course and are therefore successful in it (a *true positive* event).
2. Students who score at or above the cutoff are, in fact, not adequately prepared for the standard course and are therefore not successful in it (a *false positive* event).

3. Students who score below the cutoff need remedial instruction and therefore would not have been successful in the standard course had they enrolled in it (a *true negative* event).

4. Students who score below the cutoff do not, in fact, need remedial instruction and therefore would have been successful in the standard course had they enrolled in it (a *false negative* event).

Events 3 and 4 are not directly observable in course placement systems, because most students who are below the cutoff score have not enrolled in the standard course and therefore do not have standard course grades. Consequently, the marginal distribution of course grades is truncated below the cutoff score. In Sawyer's (1989, 1996) methodology, the proportions of true and false negatives (Events 3 and 4) are estimated by extrapolating a logistic regression function, which is estimated from the data of students who completed the standard course, to test scores below the cutoff score. Because a binary outcome variable (success, failure) is modeled as a function of test score, the logistic regression function yields an estimated conditional probability of success in the standard course:

$$\hat{P}\,[Success \mid X = x] = (1 + e^{-(a+bx)})^{-1}\,, \qquad (1)$$

where x is a particular value of the test score X, and where a and b are estimates of the model parameters α and β.

Placement system validation relies, in part, on evaluating the proportion of students correctly placed, given the cutoff score used for placement. The estimated proportion of correct placement decisions is known as the *accuracy rate* ($\hat{A}$), and is defined as the sum of the estimated proportions of true positives and true negatives.

Another statistic that may be used to evaluate a placement system is the estimated *success rate* ($\hat{S}$), which represents the proportion of students succeeding in the standard course, among all students who could have been placed in that course. $\hat{S}$ is defined as the proportion of true positives divided by the sum of the proportions of true and false positives.

Estimated $\hat{A}$ and $\hat{S}$ are functions of the estimated conditional probabilities and the marginal distribution of the predictor variable (e.g., test score) in the relevant population. For example, the proportion of true negatives can be estimated as:

$$\hat{P}\ [Failure,\ X < x_0] = \sum_{X<x_0} 1 - \hat{P}\ [Success \mid X = x_0] * n(x)\ /\ N$$

for a particular cutoff score x_0, where $n(x)$ is the number of students with a test score of x, and N is the total number of students.

Research by ACT staff indicates that truncation of course placement system data affects the accuracy of the estimated conditional probability of success (denoted $\hat{P}$ for purposes of simplification), $\hat{A}$, and $\hat{S}$. For example, Houston (1993) examined the effects of truncation on $\hat{P}$ using simulated data. Houston's findings indicate that if 25% or less truncation occurs (i.e., 25% or fewer students score below the cutoff and do not enroll in the standard course), then reasonably accurate estimates of this statistic can be obtained. Crouse (1996) used a bootstrap method to estimate confidence intervals for validity statistics. She found that relatively large samples with less truncation yield relatively more accurate estimates of $\hat{P}$, $\hat{A}$, and optimal cutoff scores[1].

Schiel and Noble (1992) used a different method to investigate the effects of truncation on estimated validity statistics, examining actual, rather than simulated, data from college-level accounting, history, psychology, and biology courses. Because all students in the sample had enrolled in standard courses, truncation had not occurred. Schiel and Noble simulated the effect of truncation on these data, and found that $\hat{A}$ and $\hat{S}$ are acceptably accurate when less than 15% truncation occurs.

The preceding truncation studies were performed under conditions in which no standard course outcome data were available for students who were below a certain point (e.g., a cutoff score) in the marginal distribution of test scores. This condition is referred to as *hard truncation*. Further research examining the effects of truncation on estimated validity statistics is important, because hard truncation is uncommon in actual placement systems. *Soft truncation*, a condition in which standard course outcomes are available for some students below a particular cutoff score, is more likely to occur. It is conceivable that validity statistics estimated under soft truncation differ in accuracy from those estimated under hard truncation.

Soft truncation can occur when cutoff scores are not strictly enforced by an institution and students below a particular cutoff choose to enroll in a standard course. There are several possible reasons for relatively low-scoring students to choose standard

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[1]The estimated optimal cutoff score may be identified by examining the $\hat{A}$ at different locations in the marginal distribution of the predictor variable. More information about this is provided in the method section.

course enrollment. For example, they may be encouraged by an adviser to enroll in a remedial course, but may instead enroll in a standard course if they are confident that they will succeed in it. Even if students below the cutoff score are less than confident of succeeding, they may still enroll in a standard course, because completing a remedial course may delay progress toward graduation and require additional tuition and fees.

Soft truncation can also occur when cutoff scores are used as guides for making placement decisions. A *decision zone,* which is an interval around a cutoff score, is one such decision-making guide. Students whose test scores are within the decision zone may be encouraged to provide additional information about their academic skills and knowledge, perhaps by taking another test. Even if the additional information indicates that these students should enroll in a remedial course, they may still choose to enroll in a standard course.

ACT has developed the Course Placement Service (CPS) to help postsecondary institutions evaluate their placement systems. Among the information provided to institutions participating in the CPS are estimated validity statistics $\hat{P}$, $\hat{A}$, and $\hat{S}$, which institutional personnel can use to identify an estimated optimal cutoff score for a particular course. Because course placement decisions have important consequences for students and institutions, accurate estimates of validity statistics are imperative, regardless of whether they are calculated by ACT or postsecondary staff. The purpose of this study, therefore, was to determine the extent to which the accuracy of these statistics is affected by soft truncation.

Data

A *placement group* consists of all students for whom a placement decision needs to be made (ACT, 1994). By definition, no truncation has occurred in the placement group. Data for six placement groups, each containing 500 observations previously simulated for use in the Crouse (1996) study, were also used in the present study. Each placement group consisted of the joint distribution of two random variables. One random variable, X, reflected the ACT Assessment score scale (1-36). The other, a binary variable Y, reflected a standard course outcome (success or failure).

The simulated data were intended to be representative of data that ACT receives from participating CPS institutions. Two factors were varied in the simulations: the slope of the logistic regression curve and the skewness of the marginal distribution of the ACT score variable X. The simulation procedure was similar to that described in Houston (1993) and involved the following steps:

1. Independently draw two random variables X_1 and X_2 from two different gamma distributions with respective α parameters a^* and b^*. Forming the ratio $X^* = X_1 / (X_1 + X_2)$ distributes X^* as a beta (a^*, b^*) random variable. Different degrees of skewness (e.g., high negative, medium negative, or zero) can be obtained by adjusting the parameter values a^* and b^*.
2. Multiply X^* by 36 and then round to the nearest positive integer.
3. Calculate the conditional probability of success using the obtained value of X^* and Equation 1, with a and b fixed to represent either a steep or flat logistic regression curve.

4. Select a random observation Y from a Bernoulli distribution with probability equal to that obtained in Step 3.

5. Repeat Steps 1-4 500 times.

The six simulated placement groups are described in Table 1. This table shows, for each placement group, the resulting skewness and logistic regression parameters a and b. The simulated data for placement Groups 1-3 had steep logistic regression curves and either high (-.62) or medium (-.29) negative skewness, or virtually zero (-.03) skewness, respectively. Groups 4-6, on the other hand, consisted of simulated data with relatively flat logistic regression curves for the three categories of skewness.

TABLE 1

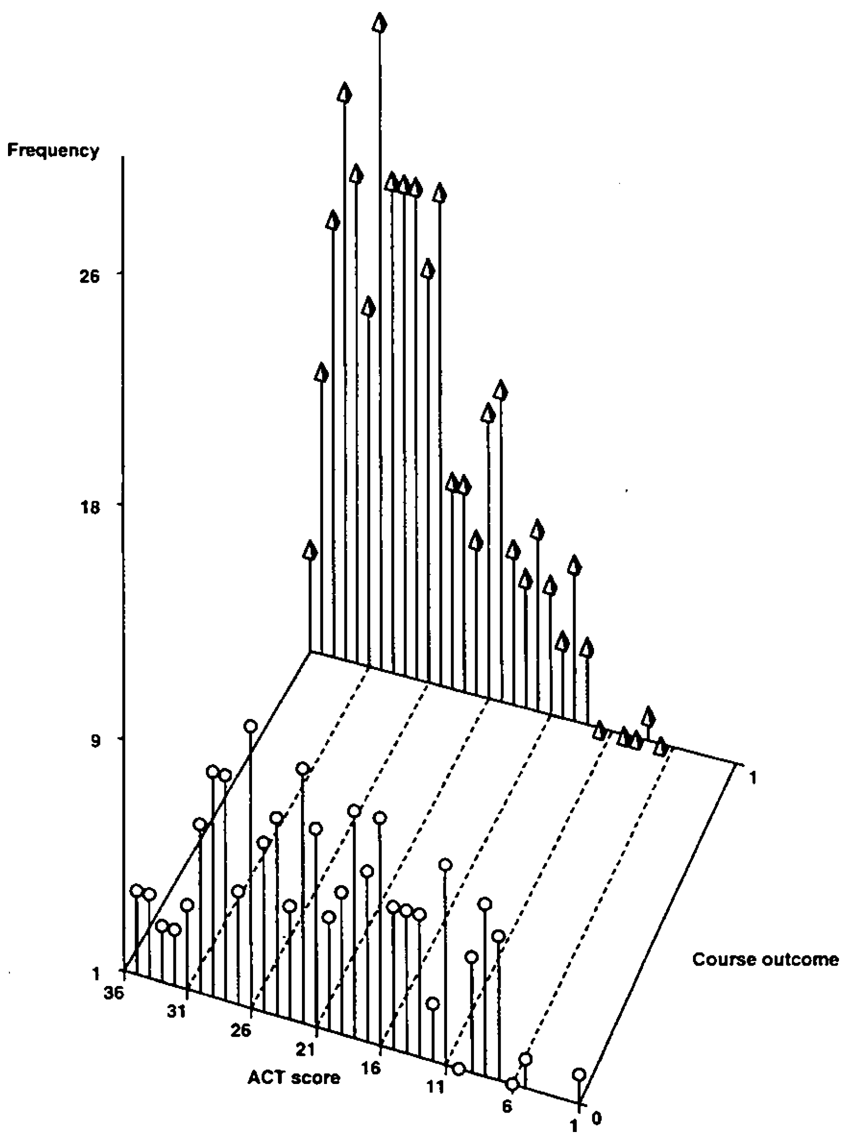
Simulated Placement Groups

Group	Slope		Skewness	
1	Steep	(a = -2.18, b = .11)	High	(-.62)
2	Steep	(a = -2.46, b = .12)	Medium	(-.29)
3	Steep	(a = -2.22, b = .11)	Zero	(.03)
4	Flat	(a = -.79, b = .03)	High	(-.61)
5	Flat	(a = -.26, b = .02)	Medium	(-.32)
6	Flat	(a = -.53, b = .03)	Zero	(-.01)

Figure 1 illustrates the simulated joint distribution of the ACT score (predictor) and course outcome variables for Placement Group 1 (steep slope, high skewness). The x-axis represents the simulated ACT score, and the y-axis represents the simulated outcome variable. Note that ACT scores are plotted in descending order from left to right. The z-axis represents the frequency of observations at a particular (x, y) coordinate point.

FIGURE 1. Joint Distribution of ACT Score and College Course Outcome

(Placement Group 1: Steep slope, high skewness)



It can be seen in Figure 1 that successful outcomes (1s) are associated with higher ACT scores. Figures A.1-A.5 in Appendix A illustrate simulated joint distributions of ACT scores and course outcomes for Placement Groups 2-6, respectively.

Method

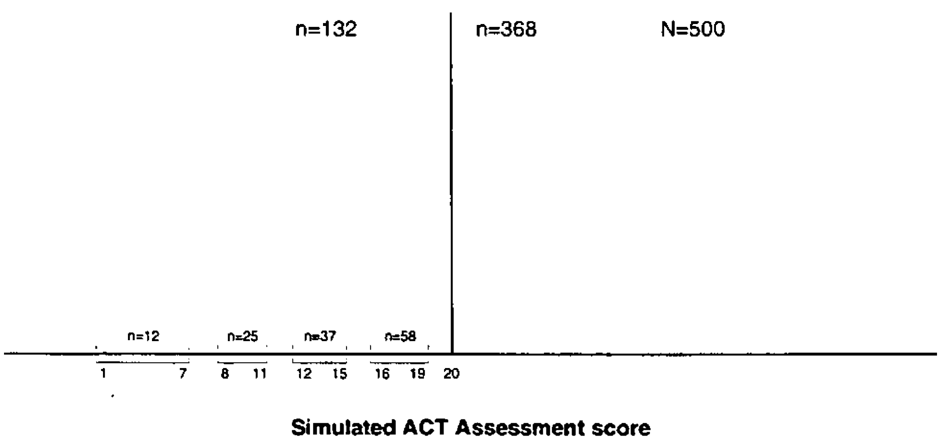
The method used to simulate soft truncation and compare the resulting estimated validity statistics to those of the (non-truncated) placement group consisted of the following steps:

1. *Calculate estimated validity statistics for the placement group.* A logistic regression function was fitted to the data of a particular placement group. The estimated conditional probabilities of success were then used together with the marginal distribution of the simulated ACT scale score to calculate estimated $\hat{A}$ and $\hat{S}$ at each scale score point.

2. *Simulate soft truncation.* An estimated optimal cutoff score typically is the score that maximizes $\hat{A}$. In addition, the maximum $\hat{A}$ corresponds to a conditional probability of success closest to .5. These facts were used to identify estimated optimal cutoff scores for placement groups and truncated placement groups, which will be referred to as *truncation samples*.

   Observations (consisting of an (x, y) pair) below the optimal cutoff score were randomly selected and then deleted from each of four different score intervals of the ACT score distribution. The four score intervals chosen for each placement group were intended to be representative of meaningful ACT score intervals. For example, very few students earn ACT scores below 8, and scores below 16 typically are considered relatively low for placement purposes. In addition, the intervals were constructed so that they contained somewhat similar percentages of observations across placement groups. Because optimal cutoffs varied by placement group, the corresponding definitions of score intervals also varied somewhat across placement groups. An illustration of the score intervals for Placement Group 1 is shown in Figure 2.

FIGURE 2. Observations Below Optimal Cutoff Score, by Score Interval
(Placement Group 1: Steep slope, high skewness)



The estimated optimal cutoff score for Placement Group 1 was 20. Of the 500 observations in this group, 132 were below the cutoff. Fifty-eight observations were in the score interval 16-19, 37 were in the score interval 12-15, 25 were in the score interval 8-11, and 12 were in the score interval 1-7. For a 20% soft truncation condition, 20% of the observations in each of the four intervals were randomly selected and then *deleted* (e.g., 7 randomly selected observations were deleted from the score interval 12-15). The resulting sample, reflecting 20% soft truncation, had 368 + (1 - .20) x 132 ≈ 474 observations.

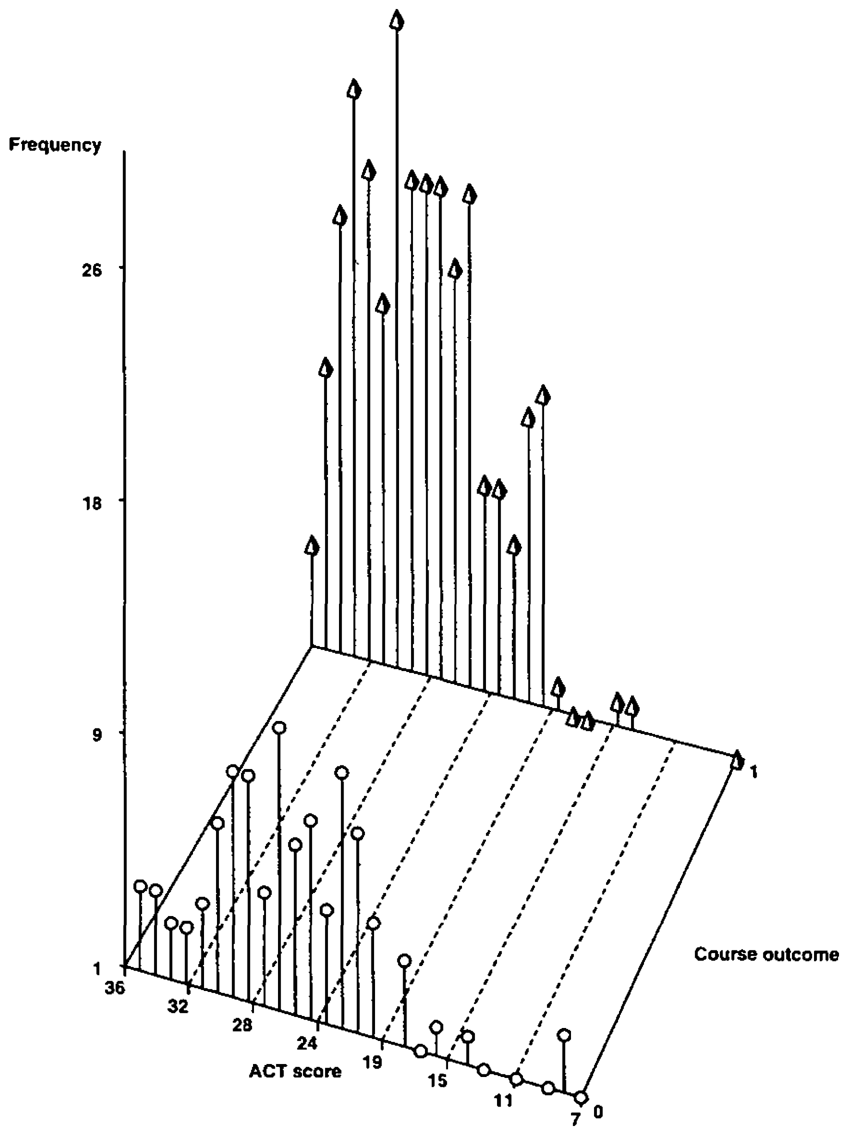
This method of simulating soft truncation was intended to ensure that some observations remained in each score interval after truncation had occurred. Otherwise, it would be possible to randomly select all of the observations for deletion from only one score interval, resulting in an oddly shaped distribution (e.g., one with no observations in score interval 12-15). Such a distribution is not likely to be encountered in an actual placement system.

The particular method chosen for simulating soft truncation could limit the generalizability of results to some degree. It is possible to simulate soft truncation in different ways. For example, instead of retaining at least a few observations in each of several test score intervals below the cutoff score, soft truncation could be simulated so that no observations below a very low score point (e.g., ACT scale score of 7 or lower) were retained. It is conceivable that different methods of simulating soft truncation could yield somewhat different estimated validity statistics.

An example of 80% soft truncation is illustrated in Figure 3, which may be compared with Figure 1 (no truncation).

FIGURE 3. Joint Distribution of ACT Score and College Course Outcome, 80% Truncated

(Placement Group 1: Steep slope, high skewness)



3. *Repeat Step 2 500 times to obtain 500 truncation samples for a particular soft truncation condition* (e.g., 20% soft truncation).

4. *Repeat Step 3 for different conditions of soft truncation.* Truncation conditions of 20%, 40%, 60%, and 80% were investigated. Note that there were 4 (soft truncation conditions) x 500 = 2000 truncation samples.

5. *Calculate estimated $\hat{P}$, $\hat{A}$, and $\hat{S}$ at each ACT scale score point (representing different hypothetical cutoff scores) for each truncation sample generated in Steps 2-4.* The ACT score marginal distribution of the placement group was used together with the $\hat{P}$s calculated from truncation samples to calculate $\hat{A}$ and $\hat{S}$.

6. *For each truncation condition, calculate median estimated $\hat{P}$, $\hat{A}$, and $\hat{S}$, across 500 truncation samples, by ACT score.*

7. *Compare the median estimates from Step 6 to those of the placement group (Step 1).*

8. *Compute estimated $\hat{P}$, $\hat{A}$, and $\hat{S}$ at each ACT scale score point for a hard truncation condition (i.e., no observations below the optimal cutoff score) and compare these statistics to those obtained in Steps 1 and 6.*

9. *Repeat the entire procedure (Steps 1-8) six times, once for each of the six simulated placement groups.*

This procedure yielded, for each combination of simulated placement group and truncation condition, estimated $\hat{P}$s, $\hat{A}$s, and $\hat{S}$s for the 36 ACT scale score points. These validity statistics were plotted for comparison purposes. In addition, differences between the validity statistics estimated from the simulated placement groups and the truncation samples were calculated. For example, the $\hat{A}$ for a (non-truncated) placement group ($\hat{A}_N$) was subtracted from the $\hat{A}$ for a 20% soft truncation condition ($\hat{A}_{20}$) for each possible ACT scale score point[2]:

$$\Delta\hat{A}_{20}^{(i)} = \hat{A}_{20}^{(i)} - \hat{A}_{N}^{(i)} \quad .$$

A total of 36 accuracy rate differences were calculated. This also pertained to the calculation of $\Delta\hat{P}_{20}$ and $\Delta\hat{S}_{20}$. These statistics were used to evaluate the accuracy of estimated $\hat{P}$, $\hat{A}$, and $\hat{S}$.

Similar calculations were performed for the 40%, 60%, 80%, and hard truncation conditions. Mean differences were then calculated, and means of the absolute value of the differences were also calculated. The mean of the absolute values of the $\Delta\hat{A}$, for example, may be expressed as

$$|\overline{\Delta\hat{A}_{20}}| = \frac{1}{36}\sum_{i=1}^{36} |\Delta\hat{A}_{20}^{(i)}| \quad .$$

Results

Estimated Conditional Probabilities of Success

The effects of soft truncation on the estimated probabilities of success for Placement Group 1 (steep slope, high skewness) are displayed in Figure 4.1. The solid curve in this figure represents probabilities for the non-truncated placement group ($\hat{P}_N$). Probabilities for the four soft truncation conditions and the hard truncation condition are shown by either dashed or dotted curves.

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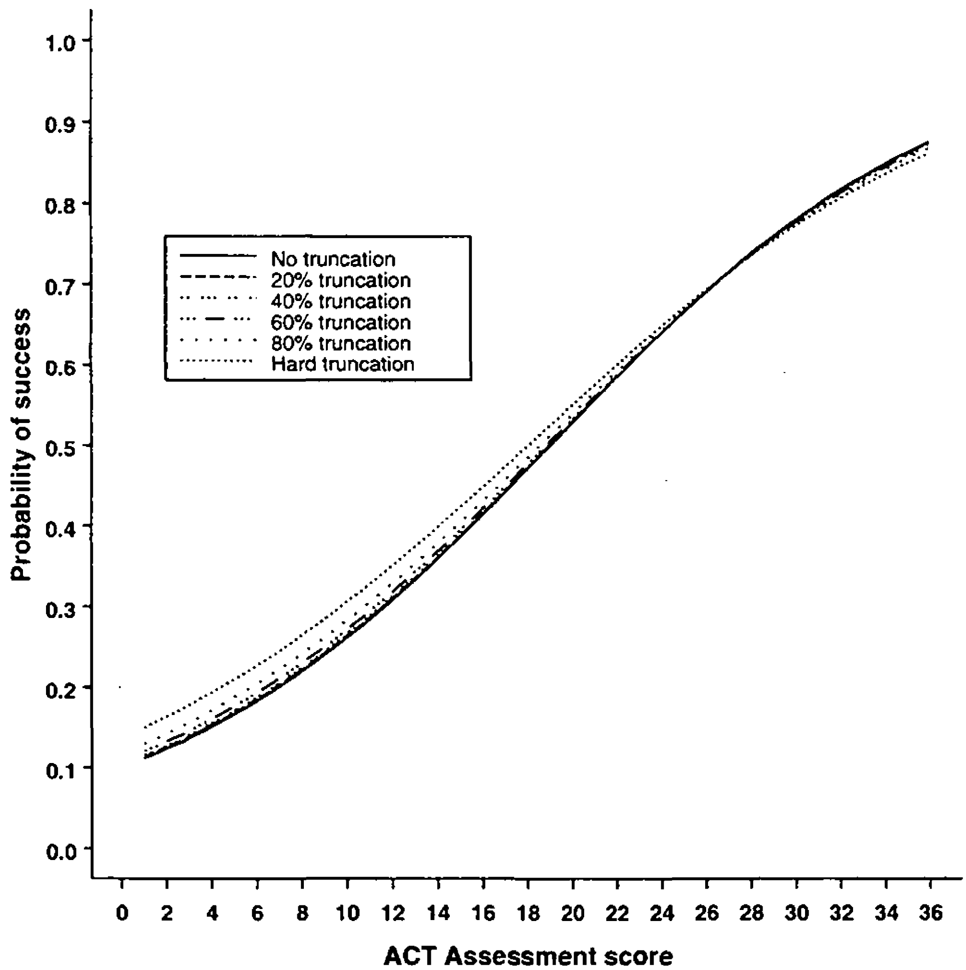
[2]A subscript of N will henceforth denote a non-truncated, simulated placement group (e.g., $\hat{S}_N$ is the success rate for this group). Numerical subscripts of 20, 40, 60, or 80 will denote the four soft truncation conditions (e.g., $\hat{P}_{60}$ is an estimated conditional probability of success obtained under the 60% soft truncation condition). A subscript of H will denote the hard truncation (i.e., 100%) condition.

FIGURE 4.1. Effects of Soft Truncation on Estimated Conditional Probability of Success

(Placement Group 1: Steep slope, high skewness)

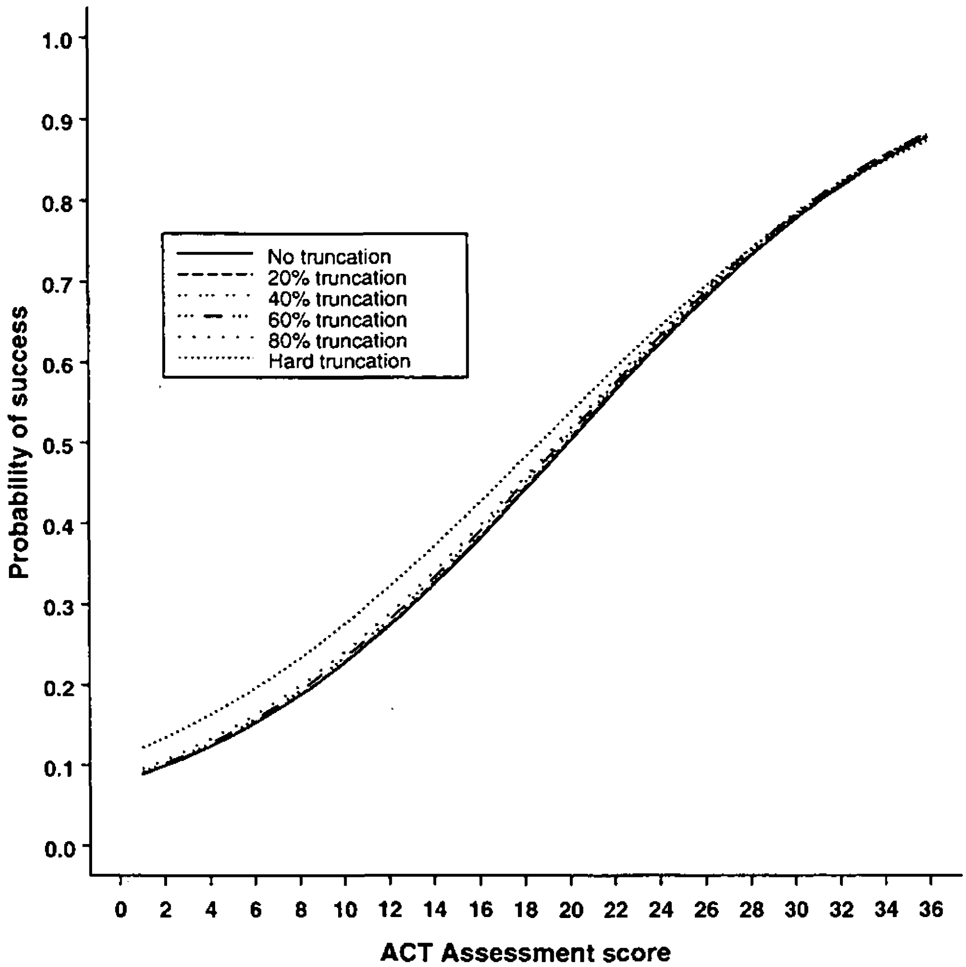


When interpreting the results in Figure 4.1, and those throughout this study, it is important to remember that probabilities based on the non-truncated placement group data are themselves estimates and are therefore subject to error. Figure 4.1 shows that the effects of soft truncation on the estimated probabilities were small, generally resulting in slight overestimates of this statistic. At higher scores (> 28), soft truncation resulted in very small underestimates of $\hat{P}_N$. The largest absolute differences between $\hat{P}_N$ and $\hat{P}$ for any of the soft truncation conditions occurred in the lower score ranges (< 15). The least accurate estimates of $\hat{P}_N$ occurred under the hard truncation condition.

The results in Figure 4.2 for Placement Group 2 (steep slope, medium skewness) are fairly similar to those of the previous figure, illustrating that soft truncation resulted in slight overestimates of $\hat{P}_N$ for this placement group. These overestimates occurred throughout the entire score range, rather than for scores of 27 or lower, as occurred for Placement Group 1.

FIGURE 4.2. Effects of Soft Truncation on Estimated Conditional Probability of Success

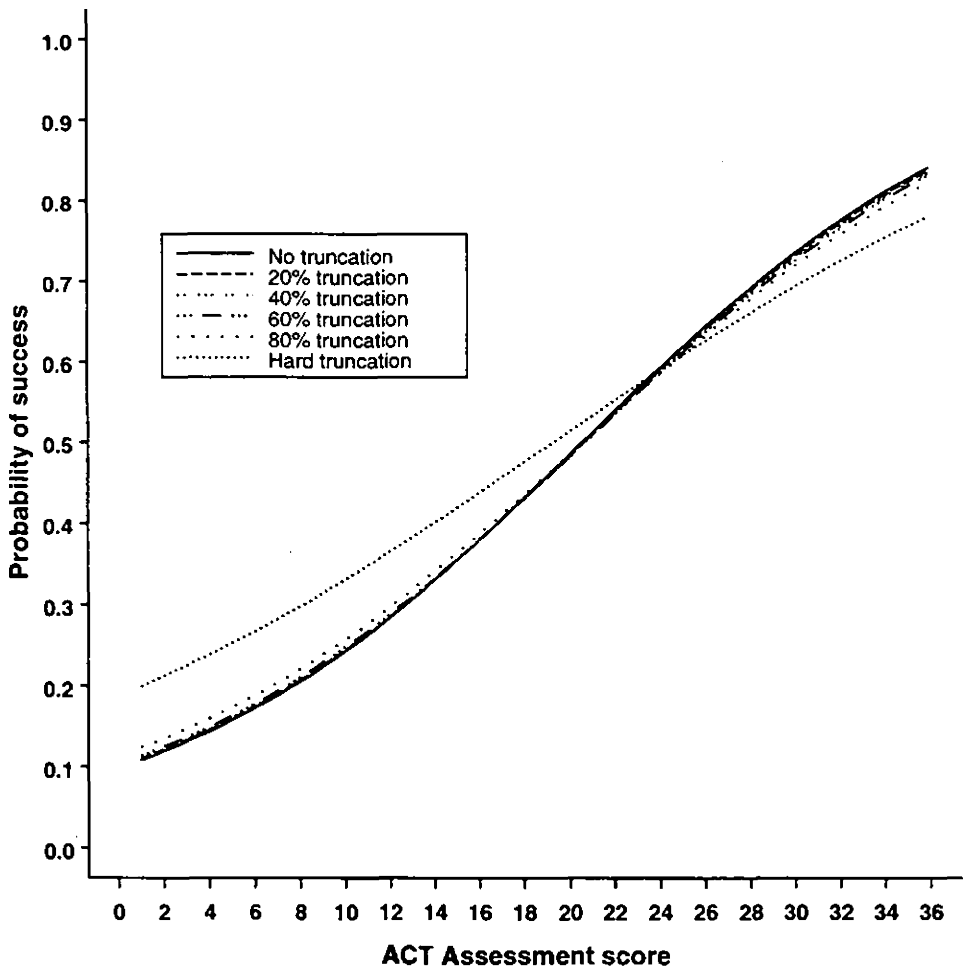
(Placement Group 2: Steep slope, medium skewness)



For Placement Group 3 (steep slope, zero skewness; Figure 4.3), both slight overestimates and underestimates of $\hat{P}_N$ resulted from soft truncation. Hard truncation resulted in less accurate estimates of $\hat{P}_N$ than those found for Placement Groups 1 and 2. This finding may be due to the fact that a relatively large number of observations are affected under hard truncation when no skewness is present in the ACT score variable. (The distributions for Placement Groups 1 and 2 were negatively skewed; consequently, there were relatively few observations in the left-hand side of these distributions that could be deleted under the hard truncation condition.)

FIGURE 4.3. Effects of Soft Truncation on Estimated Conditional Probability of Success

(Placement Group 3: Steep slope, zero skewness)



The results in Figures 4.1-4.3 suggest that when the logistic curve was steep, soft truncation had very little effect on estimating probabilities of success. In addition, negatively skewed placement group distributions were somewhat more resistant to the effects of hard truncation than were non-skewed distributions, with respect to estimating $\hat{P}_N$. Skewness did not appear to have much effect on the accuracy of estimated $\hat{P}_N$ under soft truncation conditions when the logistic curve was steep.

Figure 4.4 illustrates the effects of soft truncation on Placement Group 4, which had a relatively flat logistic curve and high negative skewness. $\hat{P}_N$ was typically overestimated under soft truncation. The estimates of $\hat{P}_N$ calculated under the soft truncation conditions were, overall, somewhat less accurate than those of the preceding three placement groups.

FIGURE 4.4. Effects of Soft Truncation on Estimated Conditional Probability of Success

(Placement Group 4: Flat slope, high skewness)

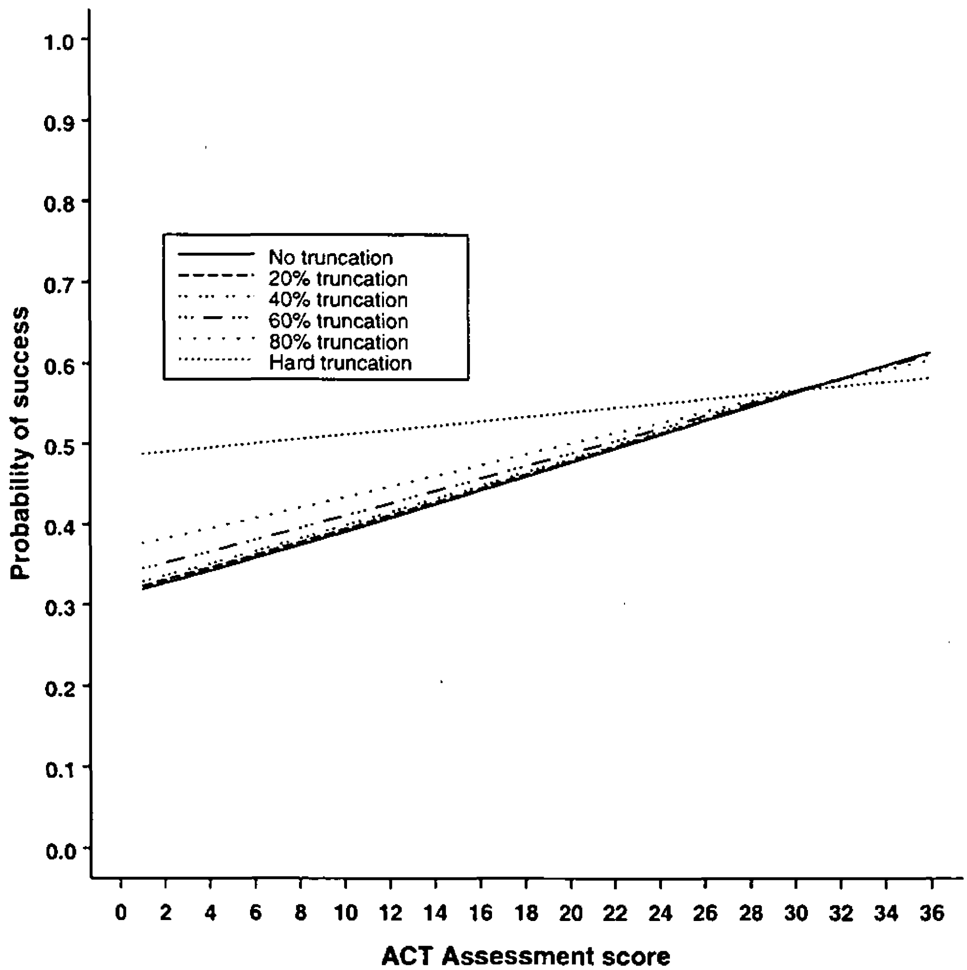
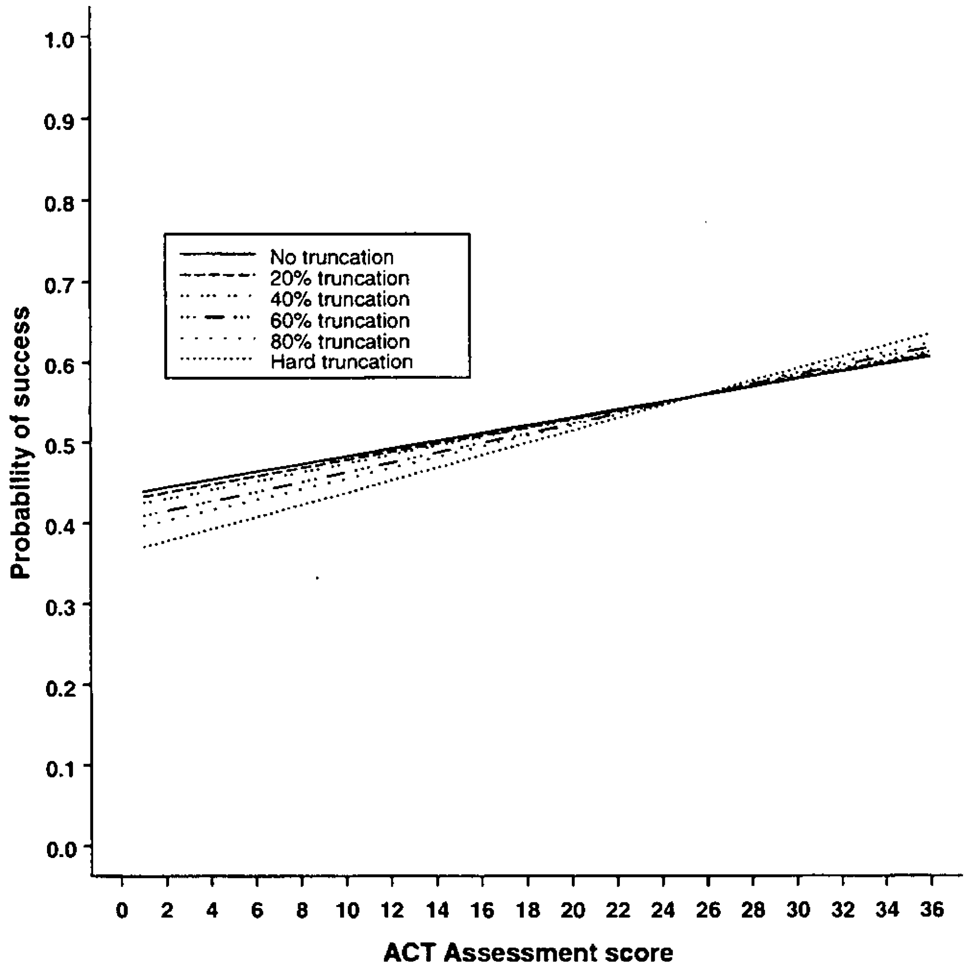


Figure 4.5 shows results for Placement Group 5 (flat slope, medium skewness). The results for this group differ from those of Placement Group 4; the probability estimates obtained under soft truncation typically underestimated, rather than overestimated, $\hat{P}_N$.

FIGURE 4.5. Effects of Soft Truncation on Estimated Conditional Probability of Success

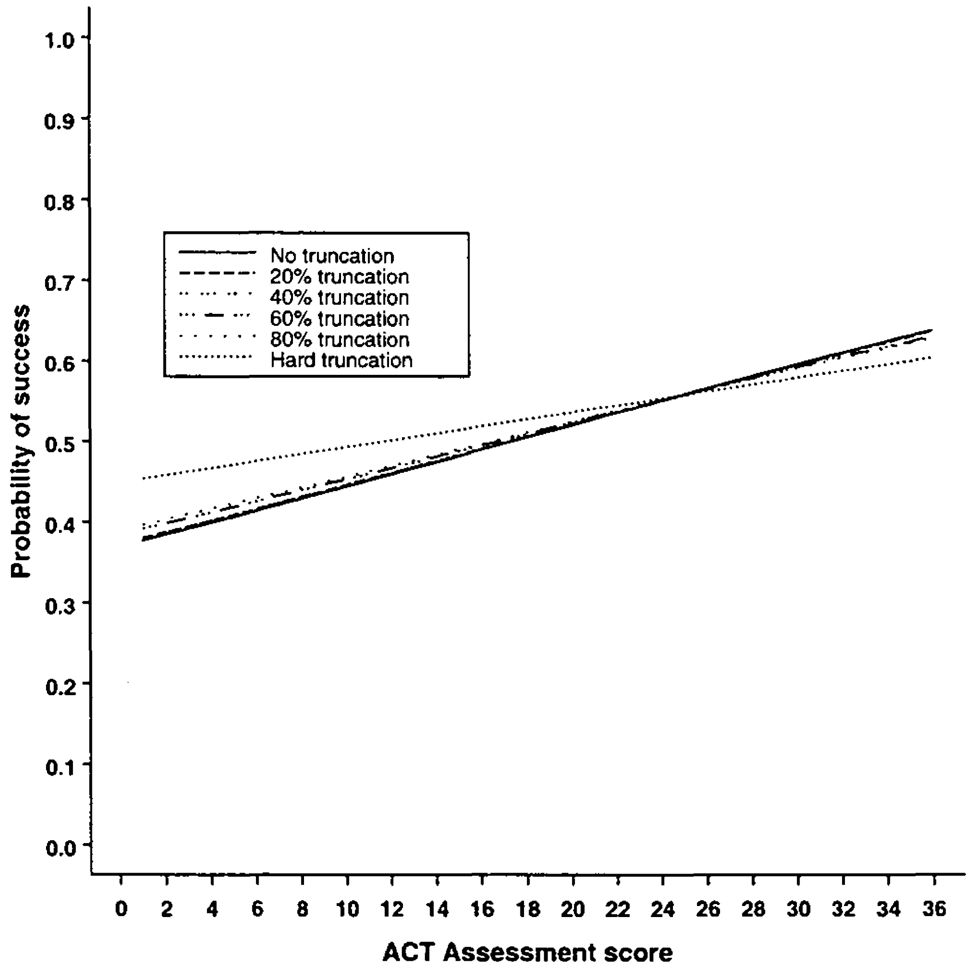
(Placement Group 5: Flat slope, medium skewness)



Results for Placement Group 6 (flat slope, zero skewness) are displayed in Figure 4.6. Estimates of $\hat{P}_N$ for this placement group were fairly accurate and comparable to those obtained when the logistic regression curve was relatively steep (Placement Groups 1-3).

FIGURE 4.6. Effects of Soft Truncation on Estimated Conditional Probability of Success

(Placement Group 6: Flat slope, zero skewness)



Figures 4.1-4.6 suggest that while soft truncation did not seriously affect estimation accuracy, relatively less accurate estimates of $\hat{P}_N$ were obtained when the logistic curve was flat, and the distribution of the predictor variable was moderately to highly skewed (i.e., Placement Groups 4 and 5). The results in Table 2 document this

observation. Mean $\Delta\hat{P}$ for each placement group, by soft truncation condition, are shown in this table. The column labeled "$\hat{P}_N$" shows the mean $\hat{P}_N$ (over 36 scale score points) for each placement group. The remaining columns show mean $\Delta\hat{P}$ and mean $|\Delta\hat{P}|$. For example, the first number beneath the column heading of "20%" (.0006) is the mean $\Delta\hat{P}$ for the 20% truncation condition in Placement Group 1. This result indicates that the average difference between $\hat{P}_{20}$ and $\hat{P}_N$, over all scale score points, was .0006.

TABLE 2

Effects of Soft Truncation on Estimated Probability of Success, by Placement Group and Truncation Condition

	Mean		Truncation				
Placement group	$\hat{P}_N$	Difference	20%	40%	60%	80%	Hard
1: Steep slope, high skewness	.4880	$\Delta\hat{P}$	.0006	.0016	.0041	.0097	.0208
		$\|\Delta\hat{P}\|$	.0012	.0027	.0060	.0127	.0256
2: Steep slope, medium skewness	.4668	$\Delta\hat{P}$	.0020	.0031	.0071	.0115	.0286
		$\|\Delta\hat{P}\|$	.0020	.0031	.0071	.0115	.0292
3: Steep slope, zero skewness	.4566	$\Delta\hat{P}$	-.0009	-.0016	-.0016	.0008	.0300
		$\|\Delta\hat{P}\|$	.0020	.0035	.0056	.0122	.0570
4: Flat slope, high skewness	.4648	$\Delta\hat{P}$	.0021	.0050	.0122	.0257	.0706
		$\|\Delta\hat{P}\|$	.0022	.0051	.0127	.0272	.0763
5: Flat slope, medium skewness	.5238	$\Delta\hat{P}$	-.0021	-.0039	-.0092	-.0124	-.0204
		$\|\Delta\hat{P}\|$	.0029	.0060	.0130	.0181	.0298
6: Flat slope, zero skewness	.5084	$\Delta\hat{P}$	.0006	.0009	.0037	.0051	.0209
		$\|\Delta\hat{P}\|$	.0012	.0014	.0064	.0085	.0328

Results for the other three soft truncation conditions, and for hard truncation, are shown in the last four columns of Table 2. Note that the signs (+, -) of the $\Delta\hat{P}$ reflect whether the probabilities obtained under truncation over- or underestimated $\hat{P}_N$. A positive value corresponds to overestimation of $\hat{P}_N$; a negative value corresponds to underestimation of this statistic.

The results in Table 2 show that the placement groups most affected by soft truncation were Groups 4 (flat slope, high skewness) and 5 (flat slope, medium skewness). Mean $|\Delta\hat{P}|$ for these groups, across all soft truncation conditions, were higher than those for the other four groups, ranging from .0022 (20% truncation, Group 4) to .0272 (80% truncation, Group 4). Mean $|\Delta\hat{P}|$ for the other placement groups ranged from .0012 (20% truncation, Groups 1 and 6) to .0127 (80% truncation, Group 1).

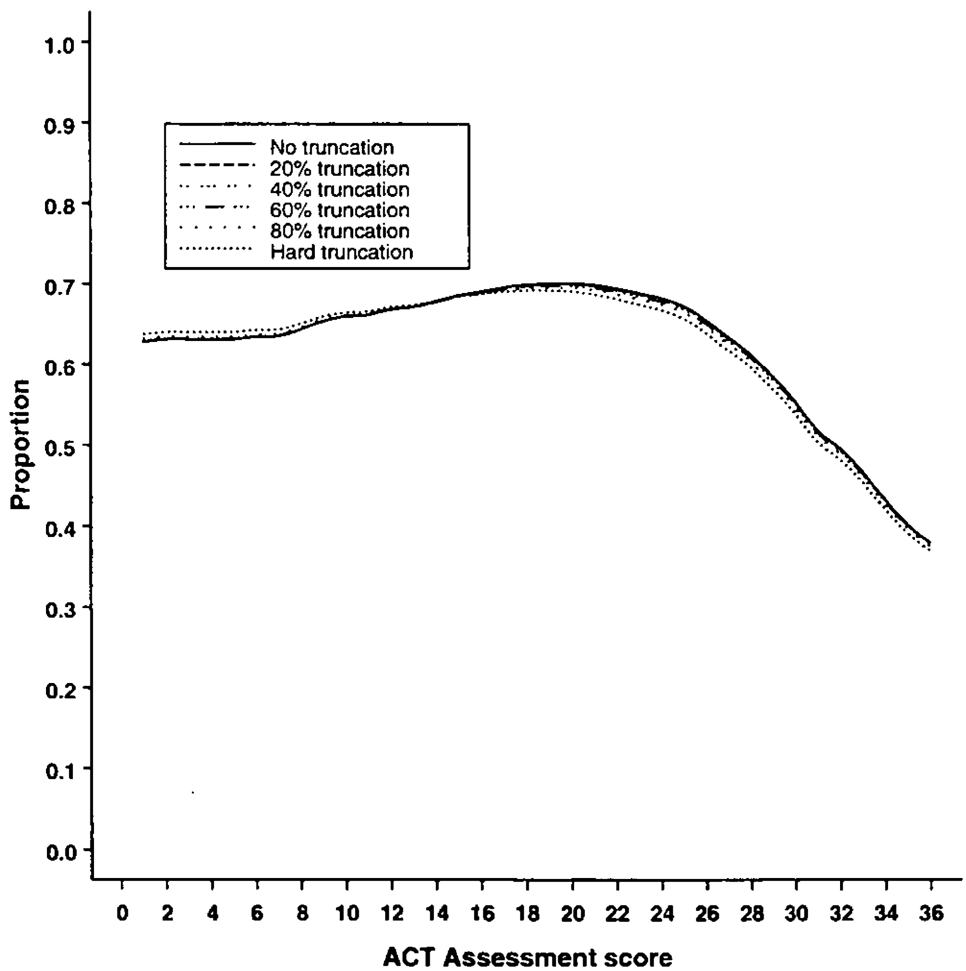
Soft truncation typically resulted in overestimates of $\hat{P}_N$; this is reflected in the positive values of mean $\Delta\hat{P}$ for Placement Groups 1, 2, 4, and 6. Group 4 was most affected by hard truncation, as indicated by its relatively large mean $|\Delta\hat{P}|$ for this condition (.0763).

Estimated Accuracy Rates and Optimal Cutoff Scores

Figure 5.1 illustrates the effects of soft truncation on estimated $\hat{A}$ for Placement Group 1. In this figure, the maximum estimated $\hat{A}_N$ corresponded to an ACT Assessment score of 20, indicating that this was the optimal cutoff score. The maximum $\hat{A}$ for the 20% and 40% soft truncation conditions were comparable, and similarly corresponded to a score of 20. For these soft truncation conditions, the "true" optimal cutoff score (20; corresponding to the maximum $\hat{A}_N$) was therefore accurately estimated. The situation was somewhat different, however, for the 60% and 80% soft truncation conditions. The maximum $\hat{A}$ occurred at a score of 19 for both of these conditions. This fact is not fully discernible in Figure 5.1, but frequency distributions of $\hat{A}$s show, for example, that the maximum $\hat{A}$ under 60% soft truncation was .6979 at a score of 19. At a score of 20, this statistic was slightly smaller (.6976). The maximum $\hat{A}_{60}$ and $\hat{A}_{80}$ both suggested that 19 was the optimal cutoff score for these respective truncation samples; the "true" optimal cutoff score was therefore underestimated by one ACT scale score point under 60% and 80% soft truncation.

FIGURE 5.1. Effects of Soft Truncation on Estimated Accuracy Rate

(Placement Group 1: Steep slope, high skewness)



Figures 5.2-5.6 show the effects of soft truncation on estimated $\hat{A}$ for the remaining five placement groups. In general, these figures indicate that relatively more accurate estimates of optimal cutoff scores were obtained when the logistic regression curve for a particular truncation sample was fairly steep. If, on the other hand, the logistic curve was relatively flat, then truncation samples with virtually no skewness of the ACT score yielded the most accurate estimates of optimal cutoff scores, regardless of the extent of soft truncation.

FIGURE 5.2. Effects of Soft Truncation on Estimated Accuracy Rate

(Placement Group 2: Steep slope, medium skewness)

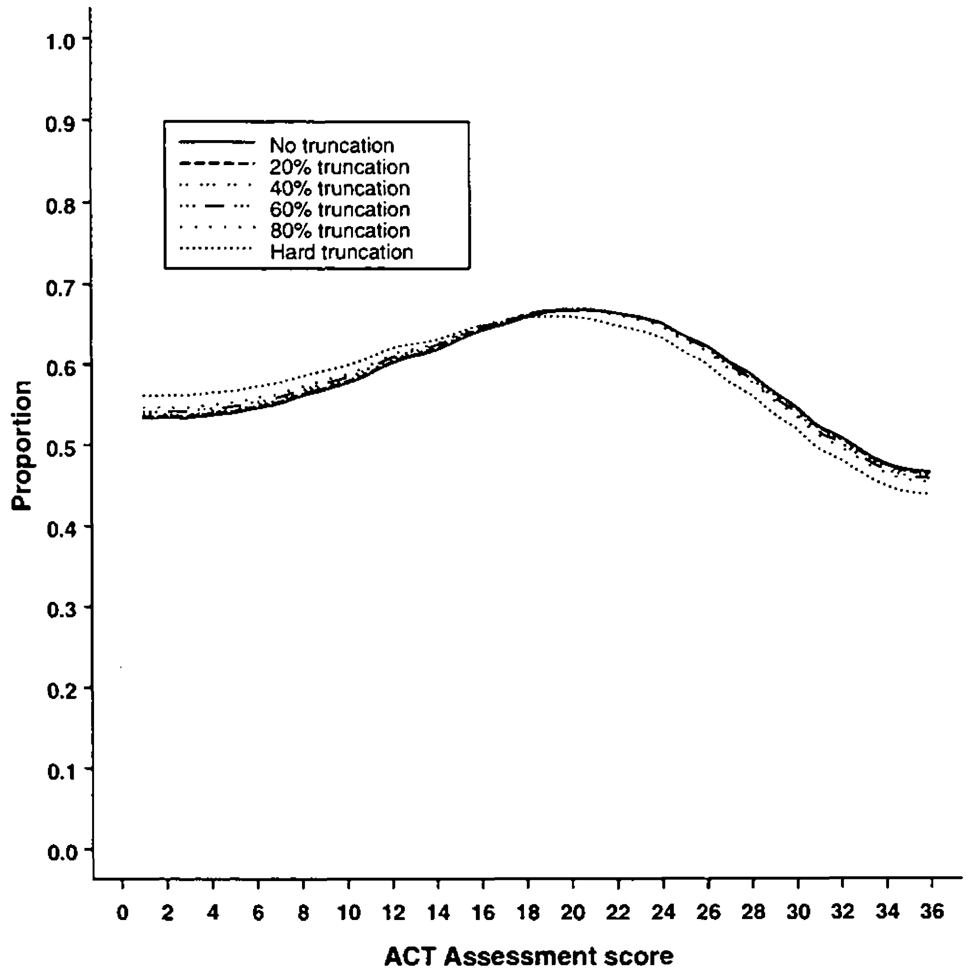


FIGURE 5.3. Effects of Soft Truncation on Estimated Accuracy Rate

(Placement Group 3: Steep slope, zero skewness)

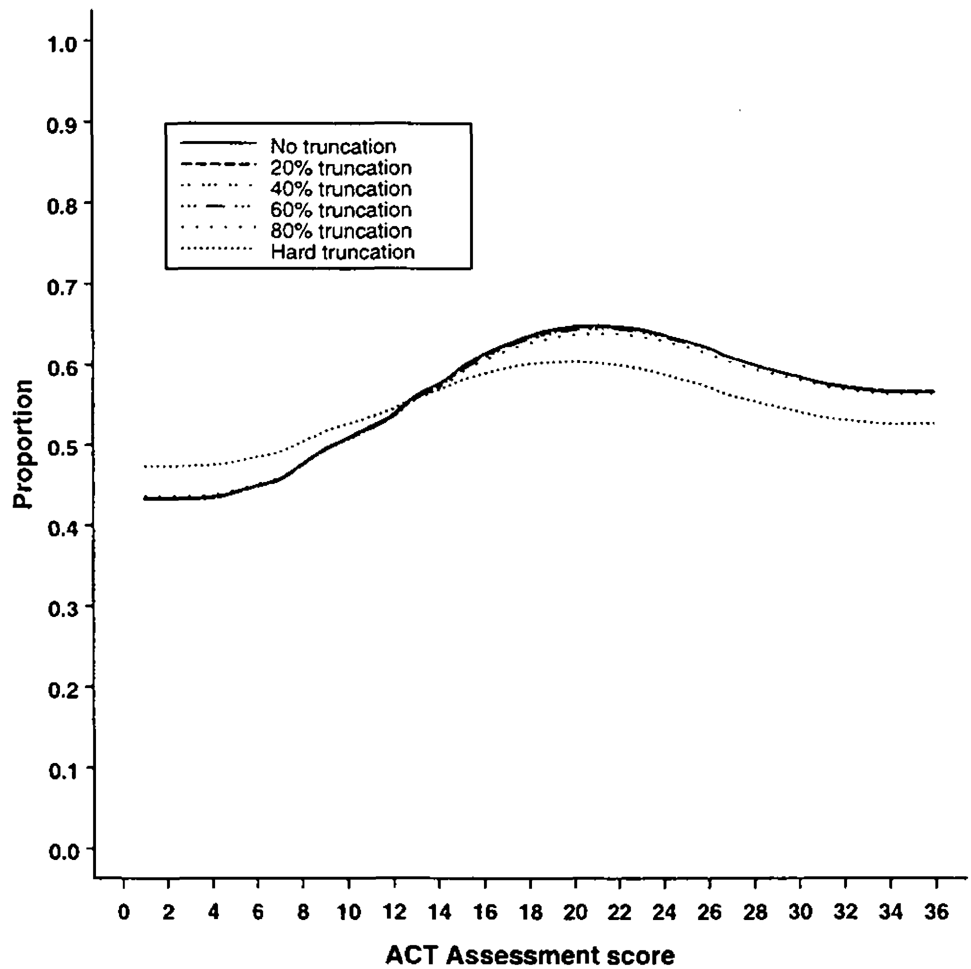


FIGURE 5.4. Effects of Soft Truncation on Estimated Accuracy Rate

(Placement Group 4: Flat slope, high skewness)

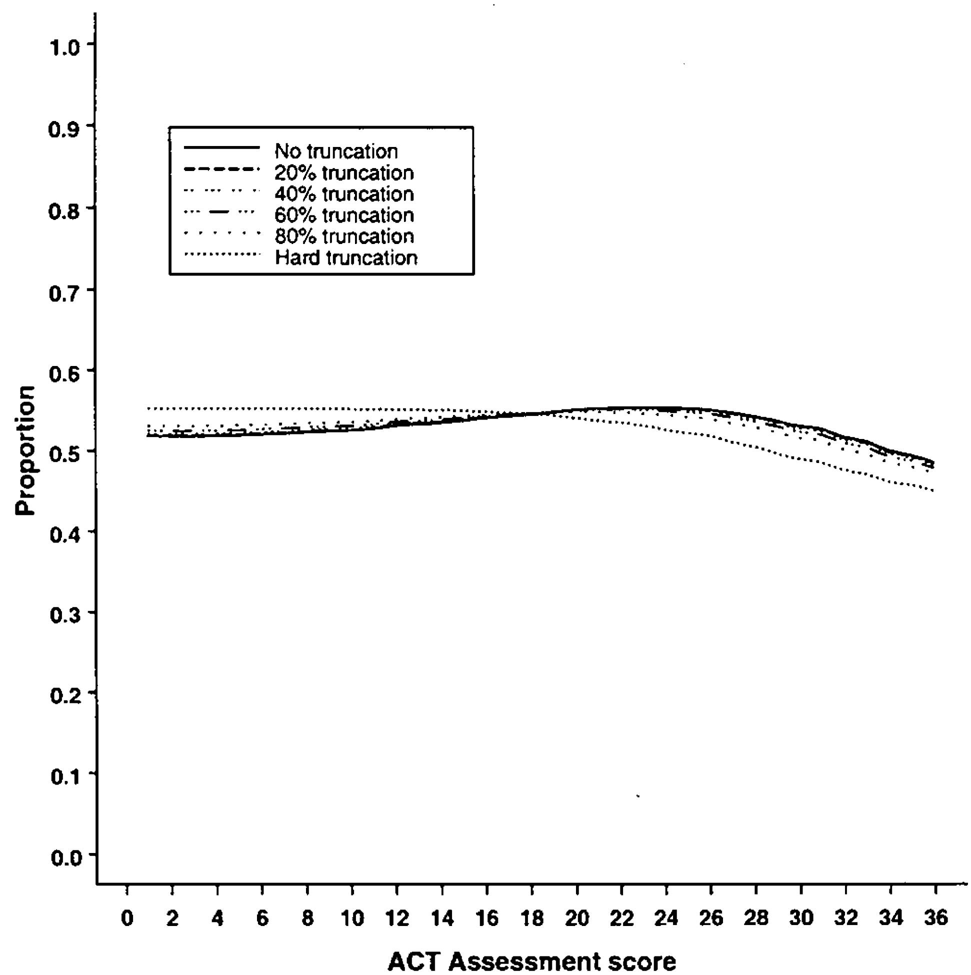


FIGURE 5.5. Effects of Soft Truncation on Estimated Accuracy Rate

(Placement Group 5: Flat slope, medium skewness)

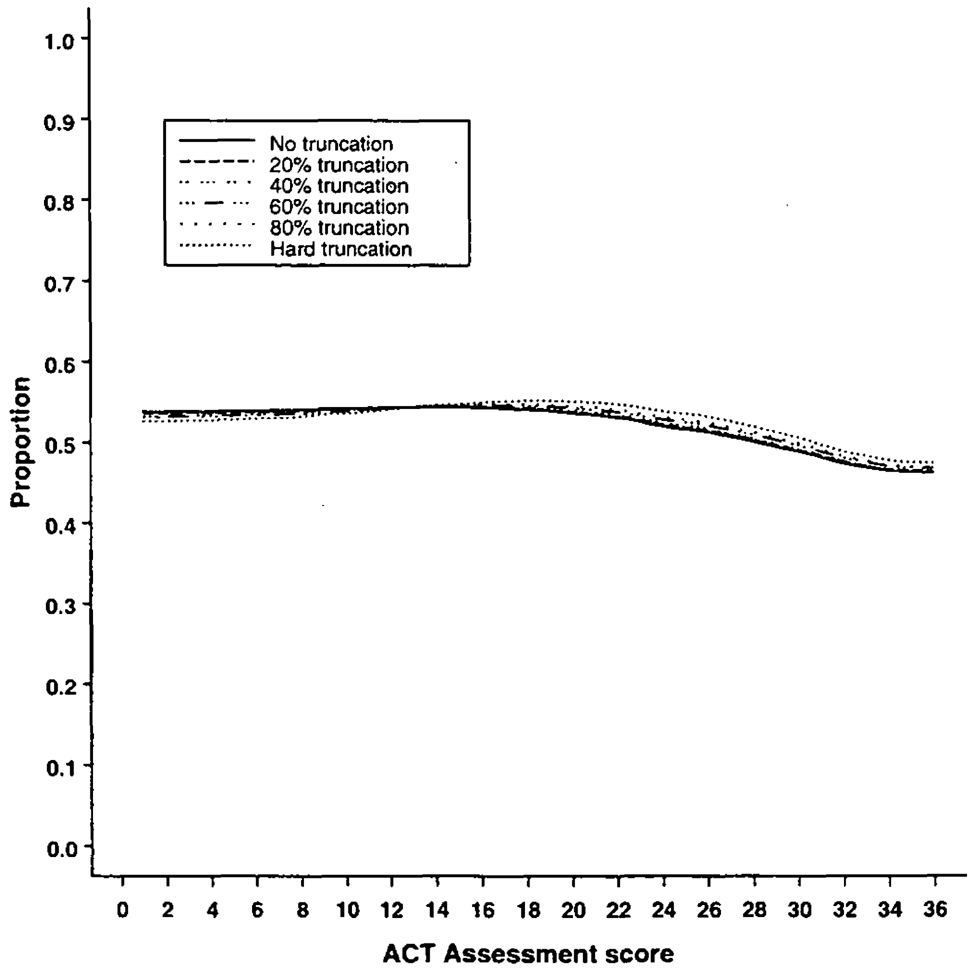
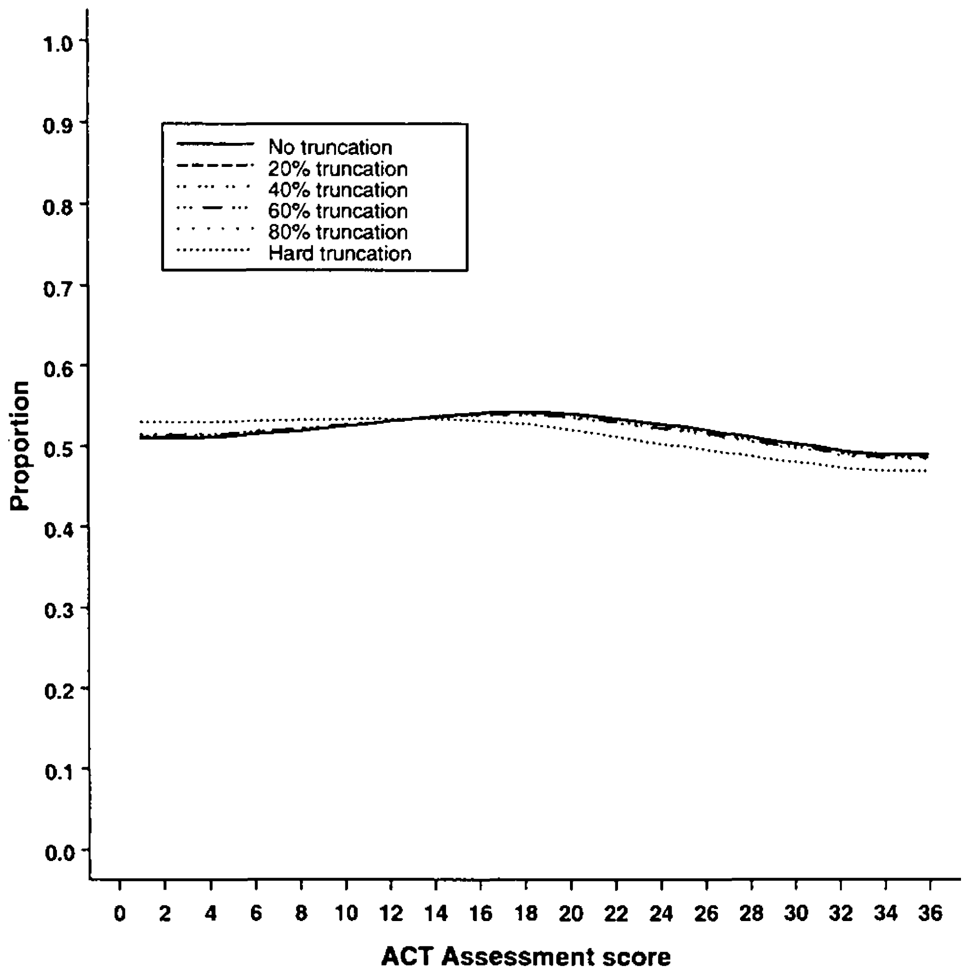


FIGURE 5.6. Effects of Soft Truncation on Estimated Accuracy Rate

(Placement Group 6: Flat slope, zero skewness)



The maximum $\hat{A}$ obtained under hard truncation yielded underestimates of the optimal cutoff score for all placement groups. In some cases, the underestimation was extreme. For example, in Figure 5.4, $\hat{A}_N$ was maximized at a score of 23, indicating that this was the "true" optimal cutoff score. In comparison, $\hat{A}_H$ was maximized at a score of 6.

Mean $\Delta\hat{A}$ and $|\Delta\hat{A}|$ are reported, by placement group, in Table 3. Relatively accurate estimates of $\hat{A}_N$ were found for Placement Groups 1 (steep slope, high skewness) and 6 (flat slope, zero skewness) across all soft truncation conditions. $|\Delta\hat{A}|$ ranged from .0004 (Group 1; 20% truncation) to .0046 (Group 1; 80% truncation) for these groups. Relatively less accurate estimates of $\hat{A}_N$ were found for Placement Groups 2 (steep slope, medium skewness), 4 (flat slope, high skewness), and 5 (flat slope, medium skewness); $|\Delta\hat{A}|$ ranged from .0009 (Group 4, 20% truncation) to .0101 (Group 4, 80% truncation) for these groups. Note that mean $\Delta\hat{S}$ and $|\Delta\hat{S}|$ are also reported in Table 3; these statistics will be examined in the following section on estimated success rates.

TABLE 3

Effects of Soft Truncation on Estimated Accuracy Rate and Success Rate, by Placement Group and Truncation Condition

Placement group	$\hat{A}_N$	$\hat{S}_N$	Mean Difference	Truncation 20%	40%	60%	80%	Hard
1: Steep slope, high skewness	.6225	.7241	$\Delta\hat{A}$	-.0004	-.0008	-.0016	-.0028	-.0047
			$\lvert\Delta\hat{A}\rvert$	.0004	.0010	.0021	.0046	.0096
			$\Delta\hat{S}$	-.0005	-.0008	-.0012	-.0012	-.0003
			$\lvert\Delta\hat{S}\rvert$	.0005	.0010	.0018	.0038	.0073
2: Steep slope, medium skewness	.5822	.6625	$\Delta\hat{A}$	.0006	.0012	.0014	.0015	-.0011
			$\lvert\Delta\hat{A}\rvert$	.0017	.0030	.0057	.0090	.0206
			$\Delta\hat{S}$	.0024	.0046	.0075	.0107	.0164
			$\lvert\Delta\hat{S}\rvert$	.0024	.0046	.0075	.0107	.0169
3: Steep slope, zero skewness	.5597	.5738	$\Delta\hat{A}$	-.0006	-.0011	-.0017	-.0038	-.0155
			$\lvert\Delta\hat{A}\rvert$	.0011	.0017	.0021	.0047	.0349
			$\Delta\hat{S}$	-.0022	-.0038	-.0050	-.0061	.0028
			$\lvert\Delta\hat{S}\rvert$	.0022	.0038	.0050	.0073	.0291
4: Flat slope, high skewness	.5307	.5495	$\Delta\hat{A}$	.0000	.0002	.0001	-.0007	-.0036
			$\lvert\Delta\hat{A}\rvert$	.0009	.0022	.0051	.0101	.0268
			$\Delta\hat{S}$	.0007	.0018	.0036	.0059	.0127
			$\lvert\Delta\hat{S}\rvert$	.0008	.0020	.0043	.0082	.0218
5: Flat slope, medium skewness	.5212	.5611	$\Delta\hat{A}$	.0005	.0013	.0026	.0037	.0061
			$\lvert\Delta\hat{A}\rvert$	.0013	.0021	.0053	.0070	.0118
			$\Delta\hat{S}$	-.0001	.0008	.0005	.0013	.0018
			$\lvert\Delta\hat{S}\rvert$	.0011	.0020	.0045	.0061	.0105
6: Flat slope, zero skewness	.5197	.5530	$\Delta\hat{A}$	-.0002	-.0003	-.0015	-.0019	-.0075
			$\lvert\Delta\hat{A}\rvert$	.0006	.0008	.0033	.0045	.0173
			$\Delta\hat{S}$	.0000	.0002	.0001	.0003	.0017
			$\lvert\Delta\hat{S}\rvert$	.0007	.0006	.0029	.0040	.0144

Even though estimates of $\hat{A}_N$ were relatively inaccurate for Placement Group 2, the corresponding estimated optimal cutoff score was acceptably accurate across soft truncation conditions (see Figure 5.2). This was not the case, however, for Placement Groups 4 and 5. Although the inaccuracy of optimal cutoff score estimates for these placement groups is illustrated in Figures 5.4 and 5.5, a clearer illustration is provided in Table 4.

The effect of truncation on the estimation of optimal cutoff scores is summarized in Table 4. For each placement group, this table shows the estimated optimal cutoff score (corresponding to the maximum $\hat{A}$) at each truncation condition. The estimated $\hat{P}$ and $\hat{S}$ are also shown. For Placement Group 1, for example, the value of $\hat{A}$ was maximized at a score of 20 when no truncation was present. The maximum $\hat{A}_N$ was .70079; $\hat{P}_N$ and $\hat{S}_N$ were .52694 and .72336, respectively. Under the 20% soft truncation condition, the optimal cutoff score was again estimated as 20 for Group 1. The corresponding maximum $\hat{A}_{20}$ was .69996 (recall that this statistic is a median calculated across 500 truncation samples). When soft truncation was at 60%, however, a cutoff of 19 was incorrectly estimated as optimal. Similar findings occurred under 80% soft truncation and under hard truncation.

TABLE 4

How Truncation Affects the Estimation of Optimal Cutoff Scores, by Placement Group

Placement group	Truncation	Optimal cutoff score	$\hat{P}$	Max($\hat{A}$)	$\hat{S}$
1: Steep slope, high skewness	None[1]	20	.52694	.70079	.72336
	20%	20	.52717	.69996	.72277
	40%	20	.52850	.69918	.72235
	60%	19	.50360	.69787	.71448
	80%	19	.51073	.69553	.71453
	Hard	19	.52384	.69192	.71604
2: Steep slope, medium skewness	None	20	.50027	.66771	.66753
	20%	20	.50280	.66862	.67006
	40%	20	.50479	.66962	.67250
	60%	20	.50994	.66952	.67564
	80%	20	.51586	.66879	.67901
	Hard	19	.50781	.66076	.67338
3: Steep slope, zero skewness	None	21	.51100	.64808	.61274
	20%	21	.50886	.64674	.61037
	40%	21	.50806	.64629	.60780
	60%	21	.50721	.64456	.60604
	80%	21	.50876	.63858	.60313
	Hard	20	.51309	.60351	.59455
4: Flat slope, high skewness	None	23	.50284	.55357	.55422
	20%	23	.50458	.55316	.55528
	40%	23	.50660	.55244	.55603
	60%	22	.50368	.55099	.55393
	80%	20	.50068	.54801	.55244
	Hard	6	.50149	.55237	.55276
5: Flat slope, medium skewness	None	14	.50236	.54379	.54845
	20%	15	.50403	.54399	.54958
	40%	15	.50124	.54466	.54916
	60%	16	.49961	.54572	.55046
	80%	17	.50196	.54739	.55304
	Hard	19	.50737	.55158	.55854
6: Flat slope, zero skewness	None	18	.50475	.54281	.54632
	20%	18	.50556	.54224	.54640
	40%	18	.50560	.54216	.54669
	60%	18	.50888	.53951	.54694
	80%	17	.50360	.53911	.54471
	Hard	12	.50133	.53478	.53842

[1]Placement group (non-truncated).

The largest difference between the optimal cutoff score estimated for a (non-truncated) placement group and one estimated for any soft truncation condition occurred for Placement Groups 4 (flat slope, high skewness) and 5 (flat slope, medium skewness). For Placement Group 4, 80% soft truncation yielded a three-point underestimate of the optimal cutoff score (23 vs. 20). For Placement Group 5, this same soft truncation condition yielded a three-point overestimate of the optimal cutoff score (14 vs. 17). The most accurate optimal cutoff score estimates occurred for Placement Groups 2 and 3, regardless of the extent of soft truncation.

Estimated Success Rates

Mean $\Delta\hat{S}$ and mean $|\Delta\hat{S}|$ are reported in Table 3. The most accurate estimates of $\hat{S}_N$ occurred for Placement Groups 1 (steep slope, high skewness) and 6 (flat slope, zero skewness). $|\Delta\hat{S}|$ ranged from .0005 (Group 1, 20% truncation) to .0040 (Group 6, 80% truncation) for these groups. Relatively less accurate estimates of $\hat{S}_N$ were found, across all soft truncation conditions, for Placement Group 2 (steep slope, medium skewness). $|\Delta\hat{S}|$ ranged from .0024 (20% truncation) to .0107 (80% truncation) for this group. The effect of soft truncation on estimated $\hat{S}$ is displayed graphically, by placement group, in Figures B.1-B.6 in Appendix B.

Discussion

Given a particular joint distribution of college course grades and placement test scores, soft truncation is likely to have very little effect on estimating conditional probabilities of success if the estimated logistic regression curve is steep. If, on the other hand, the logistic regression curve is relatively flat, then samples with no skewness in the marginal distribution of the placement test score will provide the most accurate estimates of these statistics when soft truncation is present.

Under soft truncation, optimal ACT Assessment cutoff scores will likely be underestimated (i.e., will be lower than the optimal cutoff score estimated when no truncation is present) by a maximum of one scale score point when a logistic regression curve is fairly steep. In the case of relatively flat logistic curves, optimal ACT Assessment cutoff scores may be under- or overestimated by a maximum of three scale score points. Accurate estimates of the conditional probability of success, accuracy rate, and optimal ACT Assessment cutoff score can likely be obtained even when 40%, and in some cases 60% or 80%, soft truncation occurs. Moreover, the slope of the logistic regression curve and the skewness of the test score marginal distribution have little to do with the relative accuracy of these statistics unless soft truncation exceeds 40%.

The findings of this study have implications for the estimated validity statistics that postsecondary institutions use to establish cutoff scores for course placement. An optimal cutoff score for a particular college-level standard course that is underestimated by one ACT Assessment scale score point may have little practical consequence for an institution and its students. For example, it is likely that relatively few students who otherwise would have been placed into a remedial course would be placed (incorrectly) into a corresponding standard course if the optimal ACT Assessment cutoff score were to be underestimated by one scale score point. In comparison, if an optimal cutoff were to be underestimated by three ACT Assessment scale score points, relatively more students would be affected.

When interpreting estimated validity statistics, it is important for postsecondary staff to evaluate course placement data with respect to skewness of the test score distribution and slope of the logistic regression curve. For example, if it were found that the logistic regression curve was relatively flat for a particular institution's data, that a moderate degree of negative skewness was present in the marginal distribution of the ACT Assessment score, and that 80% of the students who scored below the institution's present cutoff score had not enrolled in and completed the standard course, then it is possible that the optimal cutoff score could be underestimated by as many as three scale score points. It may be possible to direct future research efforts on truncation toward developing some type of correction for validity statistics estimated under the conditions described in this example. Suitable corrections would likely permit postsecondary institutions to derive greater benefit from using estimated validity statistics to evaluate their placement systems and establish cutoff scores.

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Appendix A

Joint Distributions of ACT Score and College Course Outcome for Placement Groups 2-5

FIGURE A.1. Joint Distribution of ACT Score and College Course Outcome

(Placement Group 2: Steep slope, medium skewness)

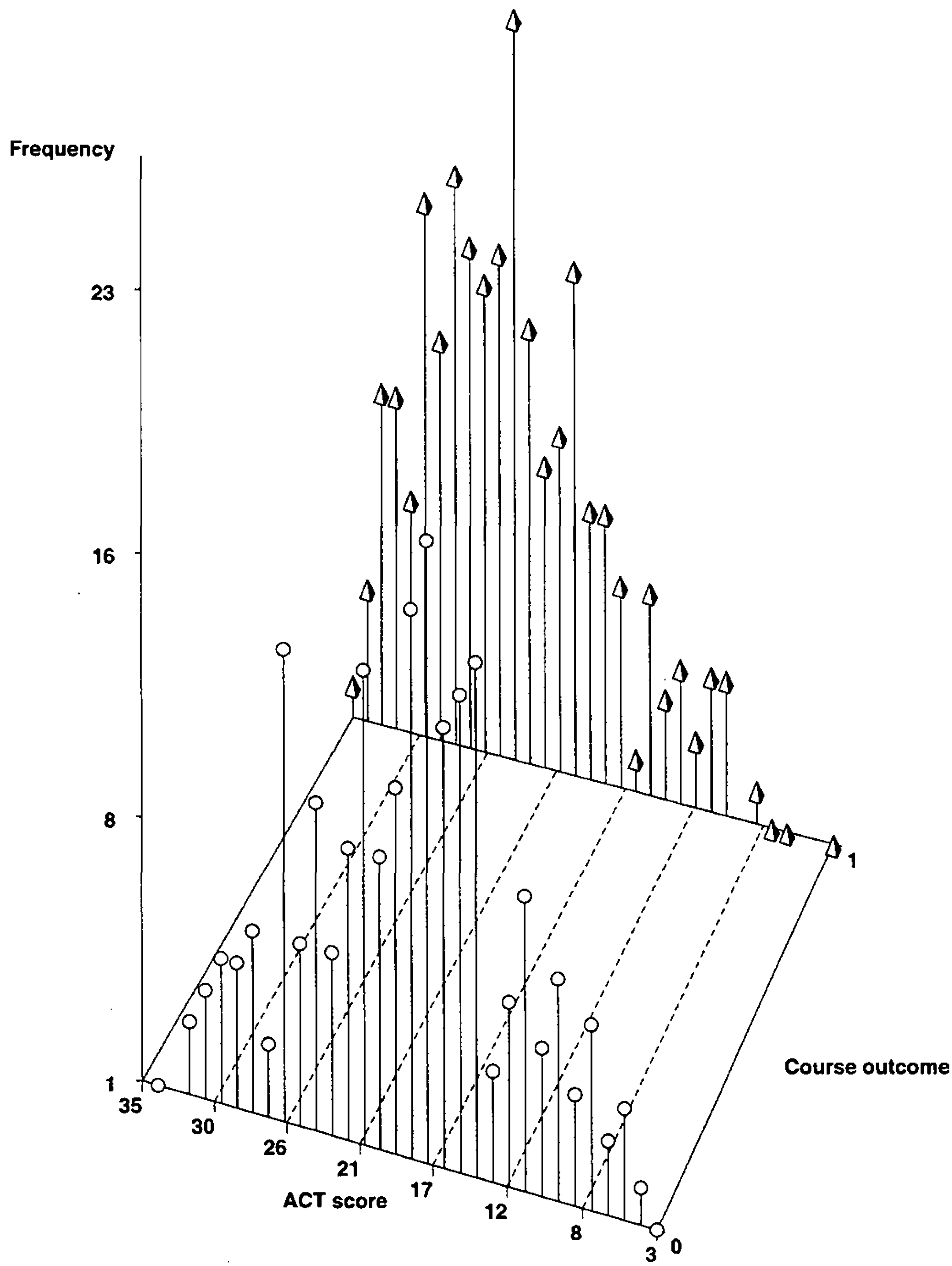


FIGURE A.2. Joint Distribution of ACT Score and College Course Outcome

(Placement Group 3: Steep slope, zero skewness)

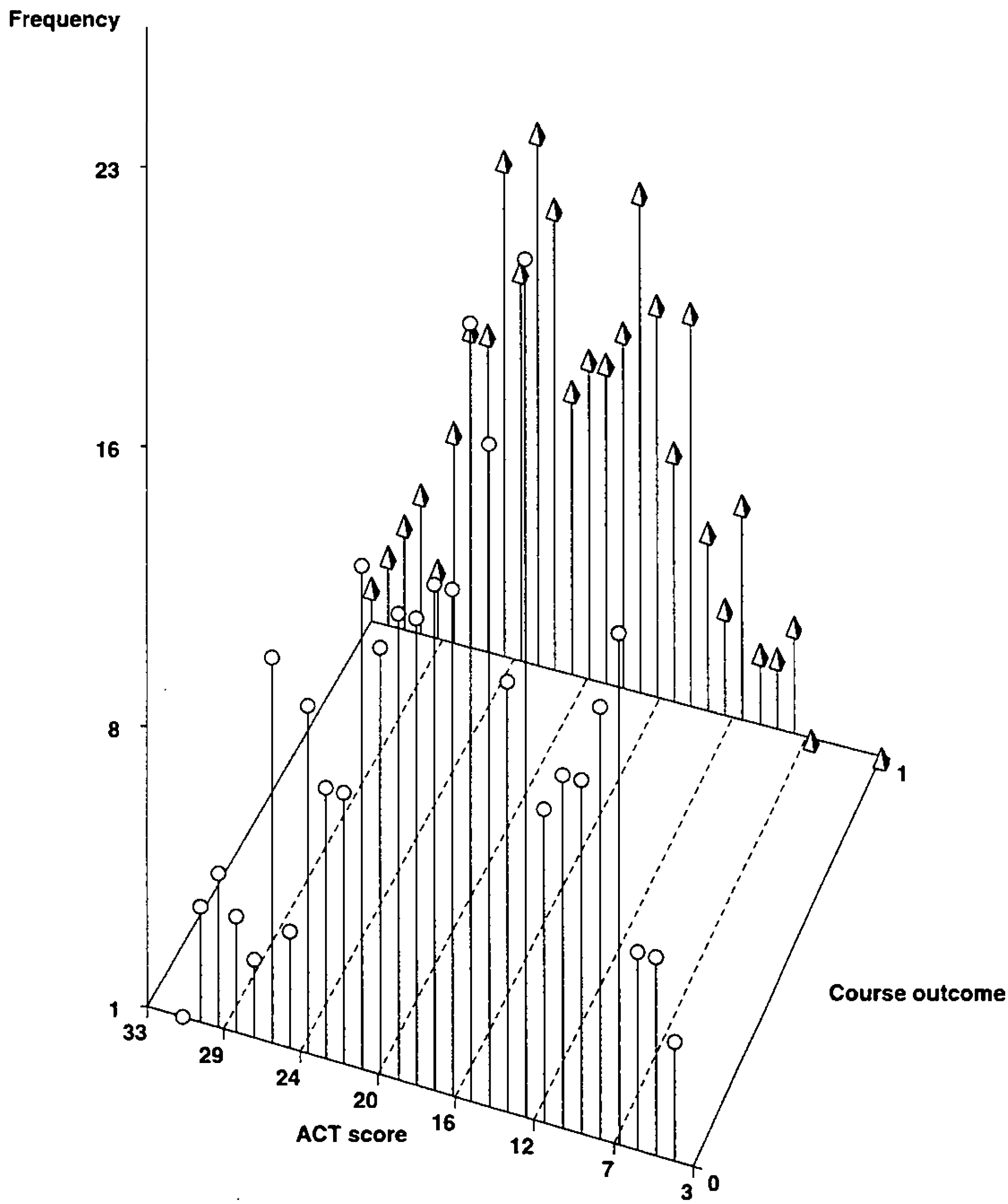


FIGURE A.3. Joint Distribution of ACT Score and College Course Outcome

(Placement Group 4: Flat slope, high skewness)

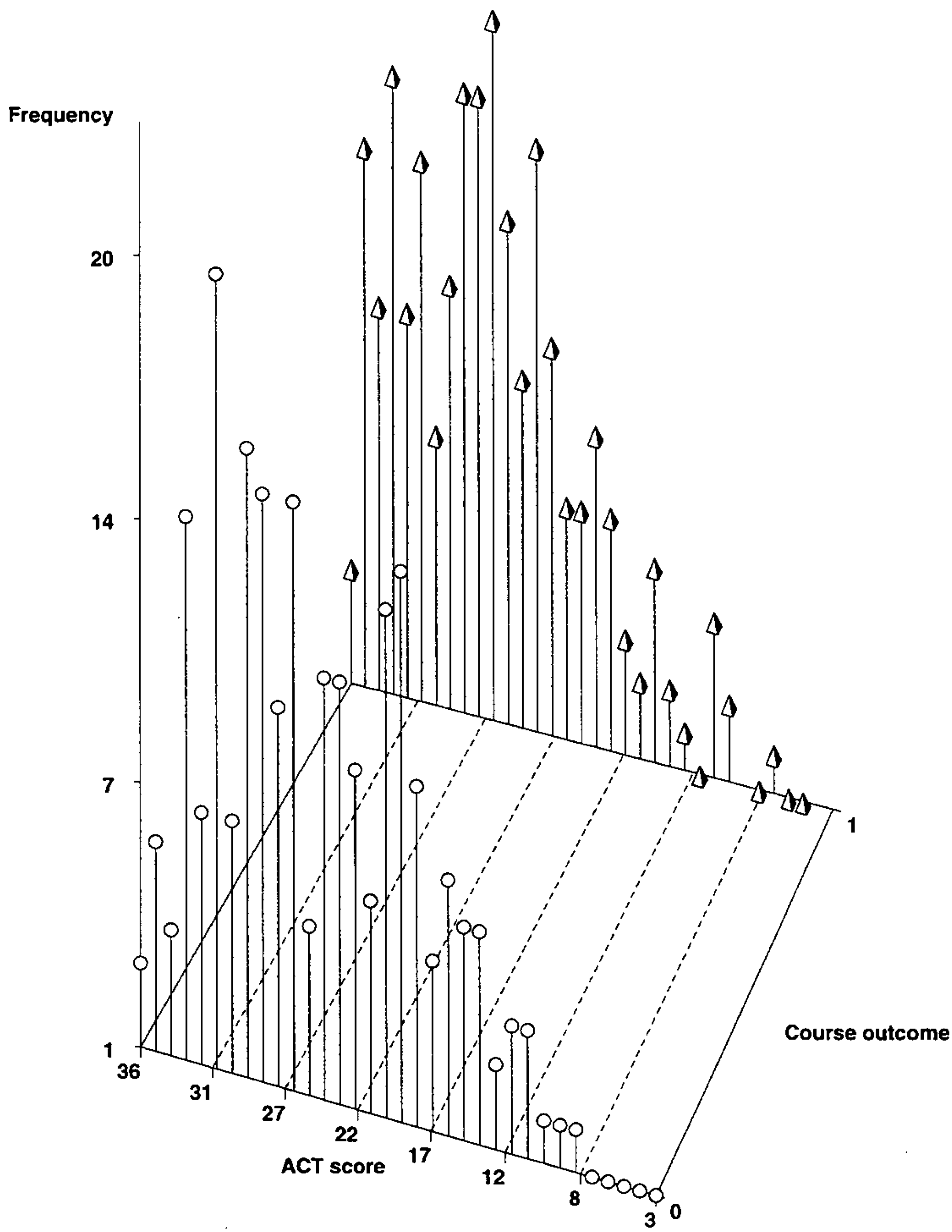


FIGURE A.4. Joint Distribution of ACT Score and College Course Outcome

(Placement Group 5: Flat slope, medium skewness)

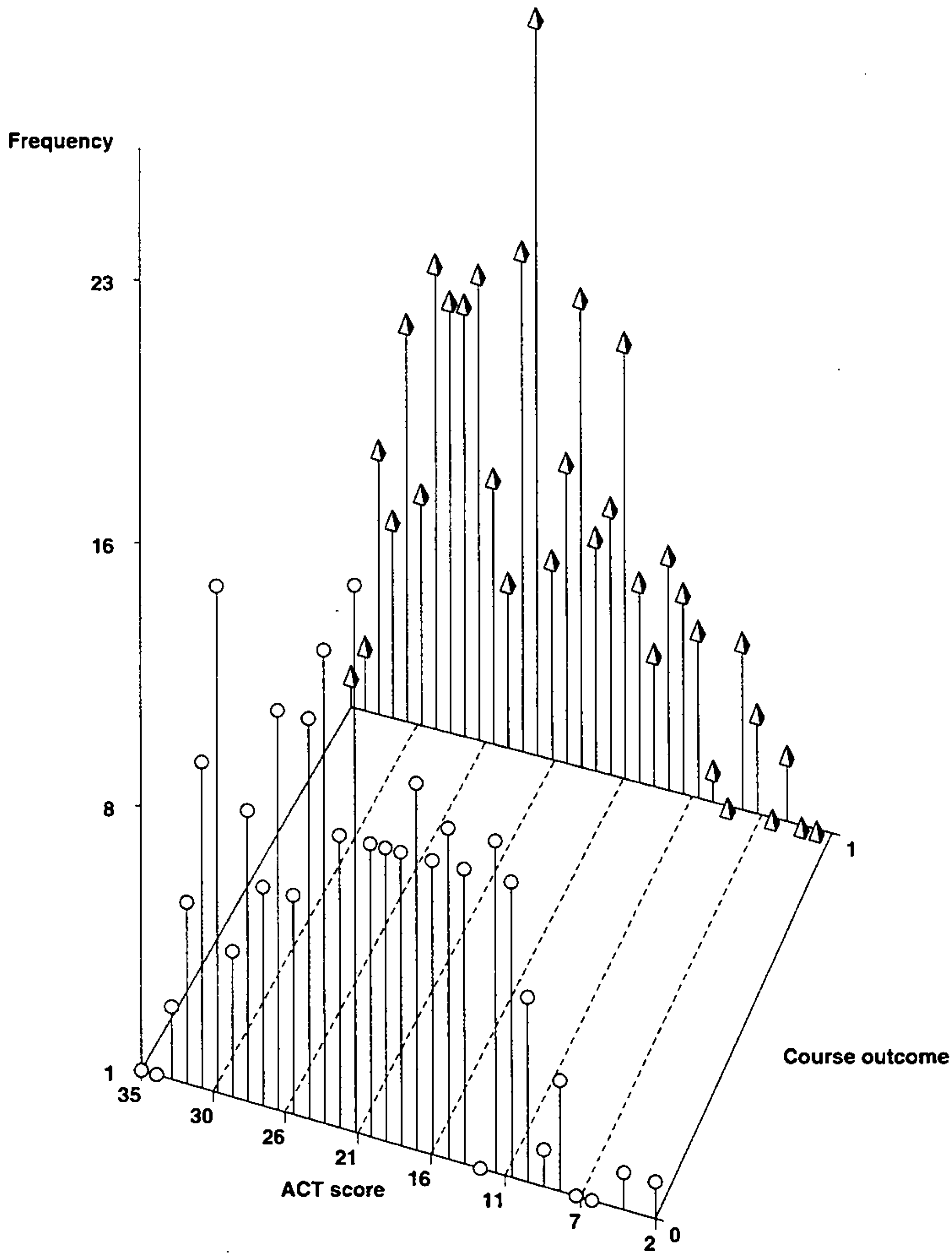
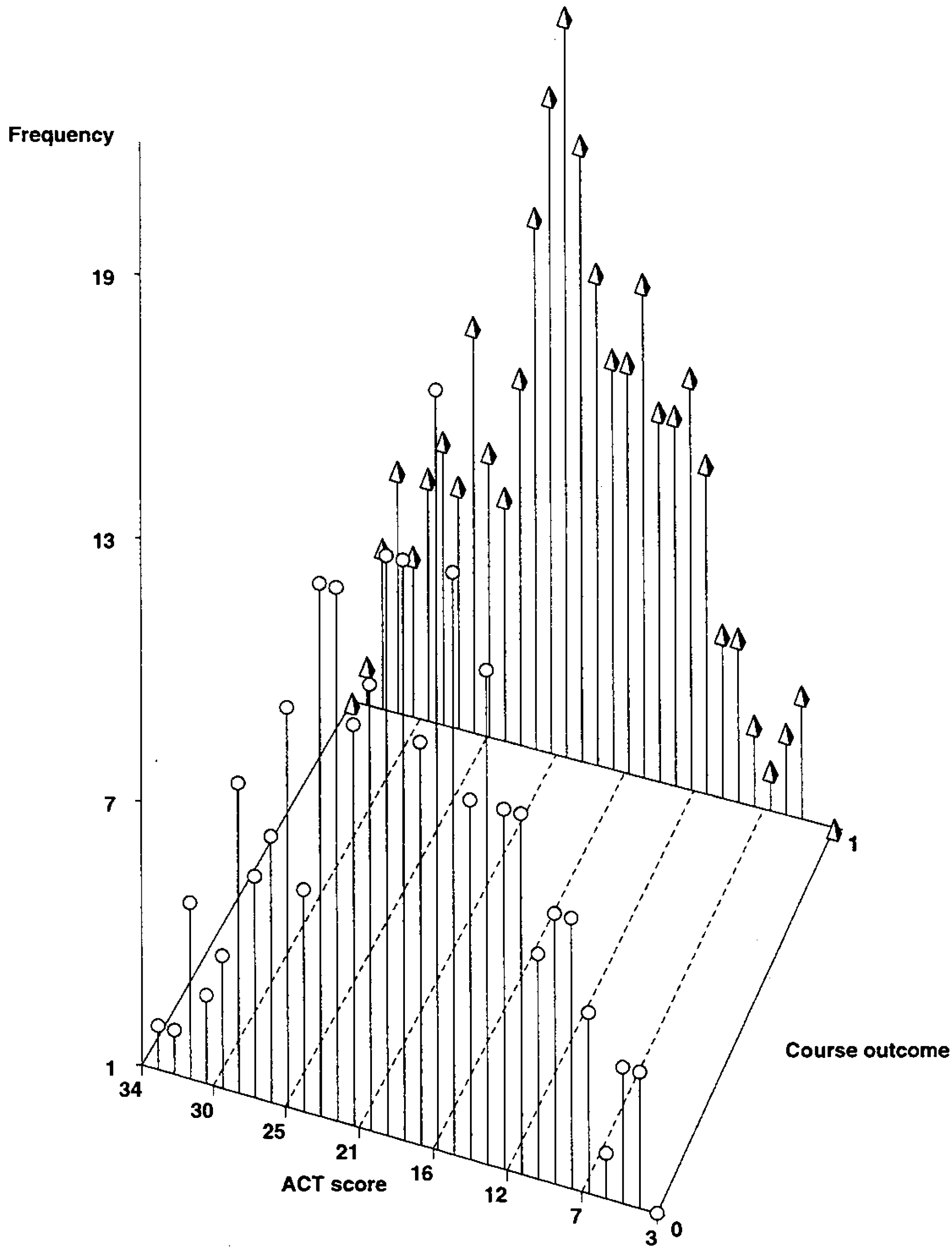


FIGURE A.5. Joint Distribution of ACT Score and College Course Outcome

(Placement Group 6: Flat slope, zero skewness)



Appendix B

Effects of Soft Truncation on Estimated Success Rate, by Placement Group

FIGURE B.1. Effects of Soft Truncation on Estimated Success Rate

(Placement Group 1: Steep slope, high skewness)

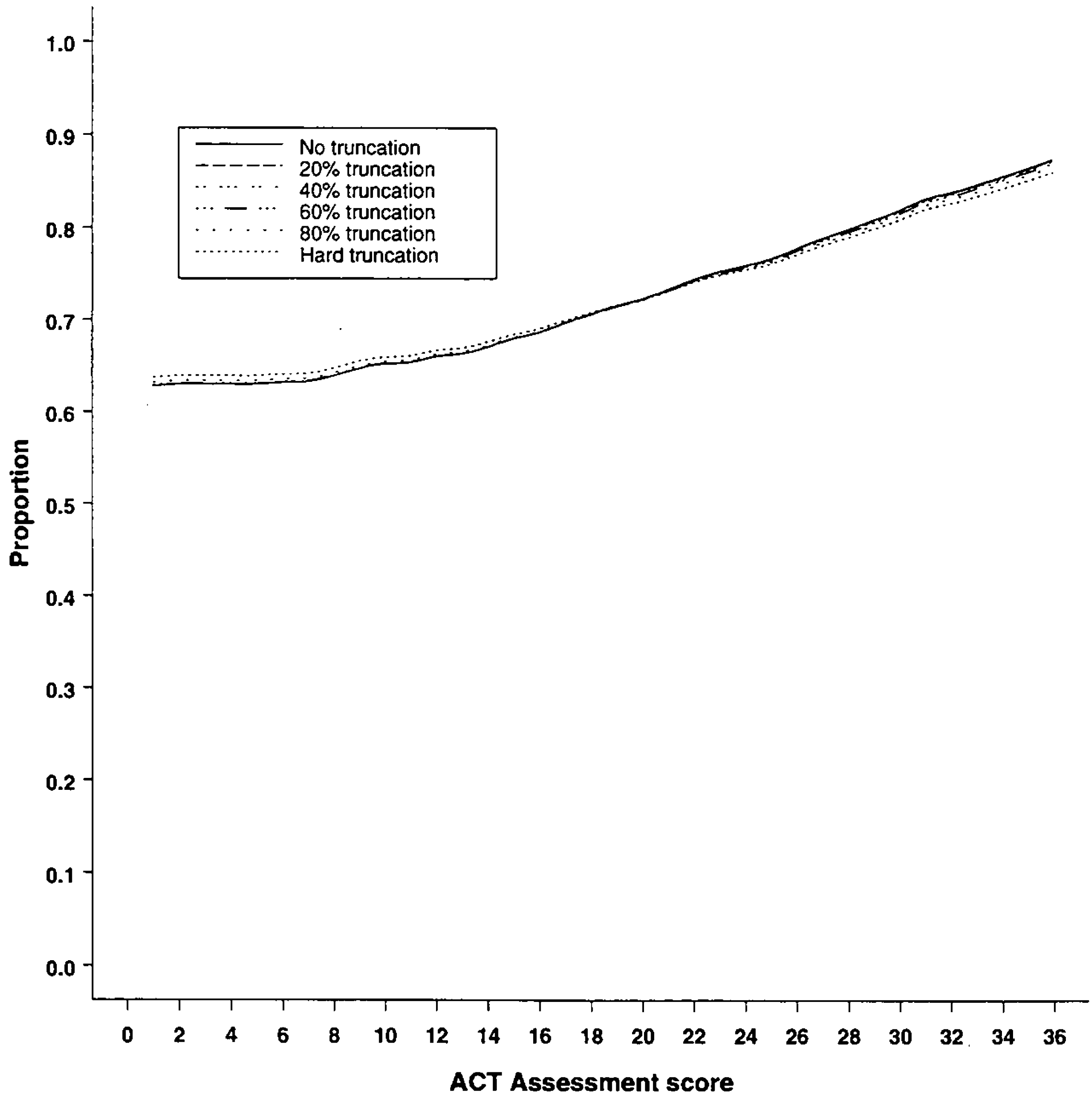


FIGURE B.2. Effects of Soft Truncation on Estimated Success Rate

(Placement Group 2: Steep slope, medium skewness)

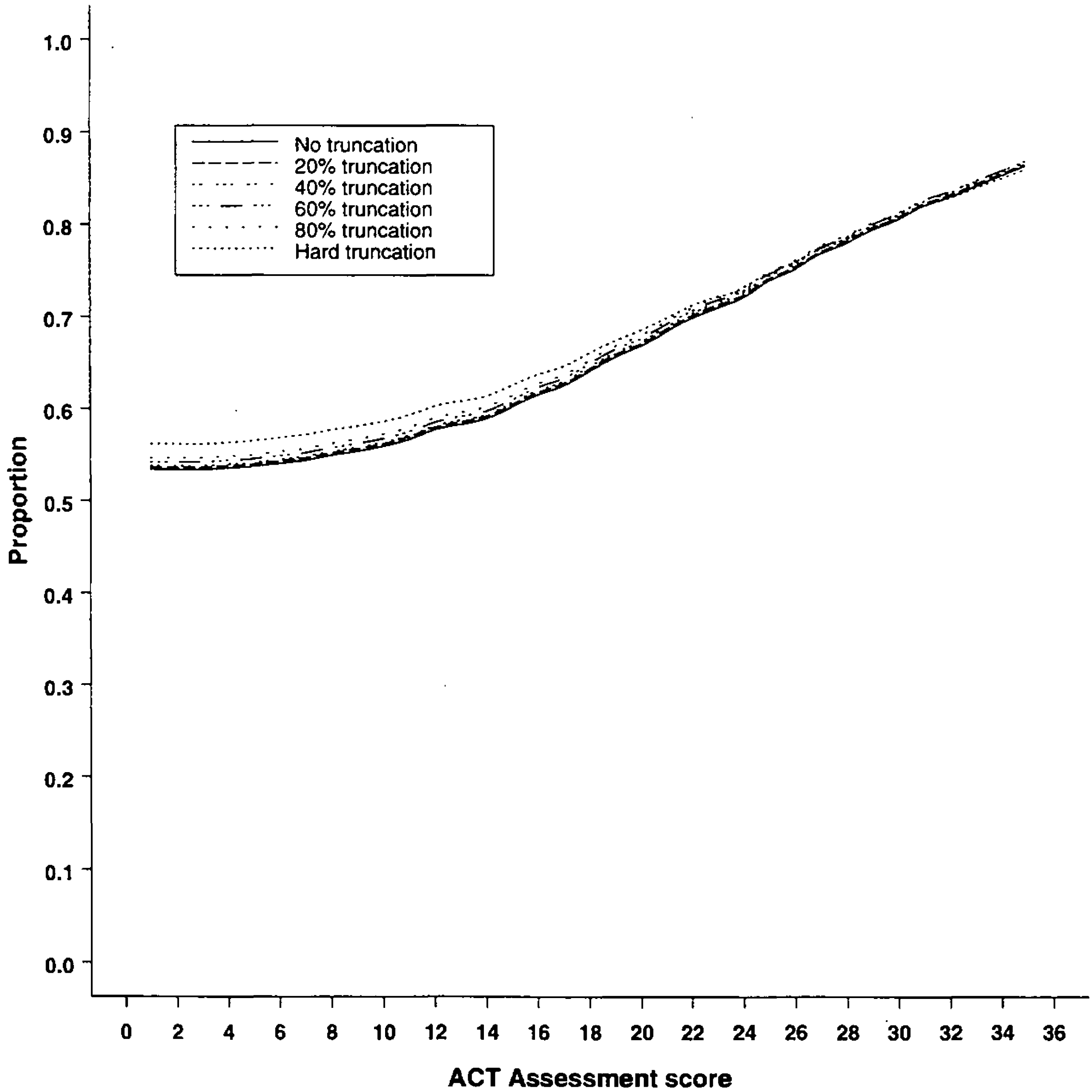


FIGURE B.3. Effects of Soft Truncation on Estimated Success Rate

(Placement Group 3: Steep slope, zero skewness)

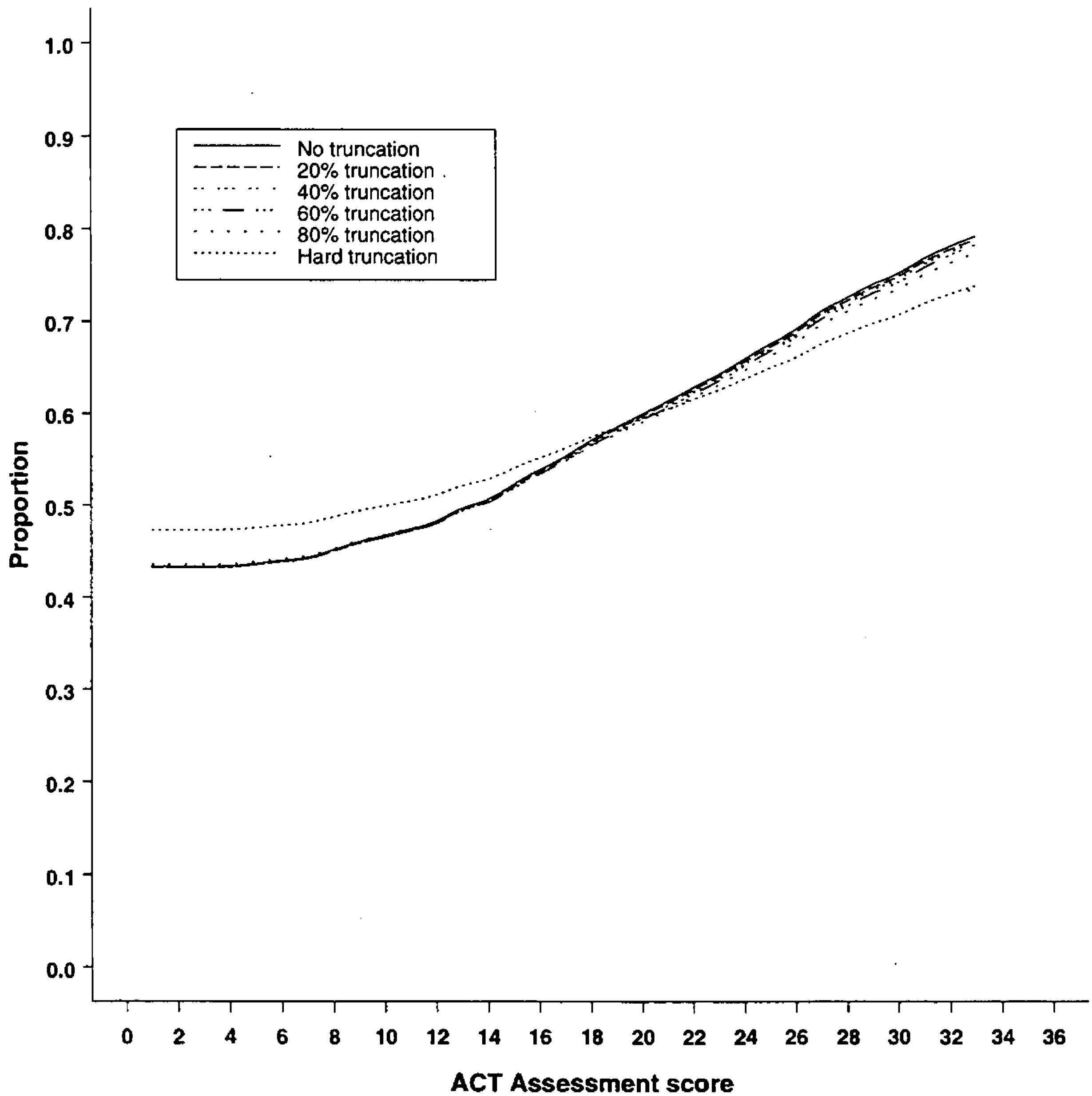


FIGURE B.4. Effects of Soft Truncation on Estimated Success Rate

(Placement Group 4: Flat slope, high skewness)

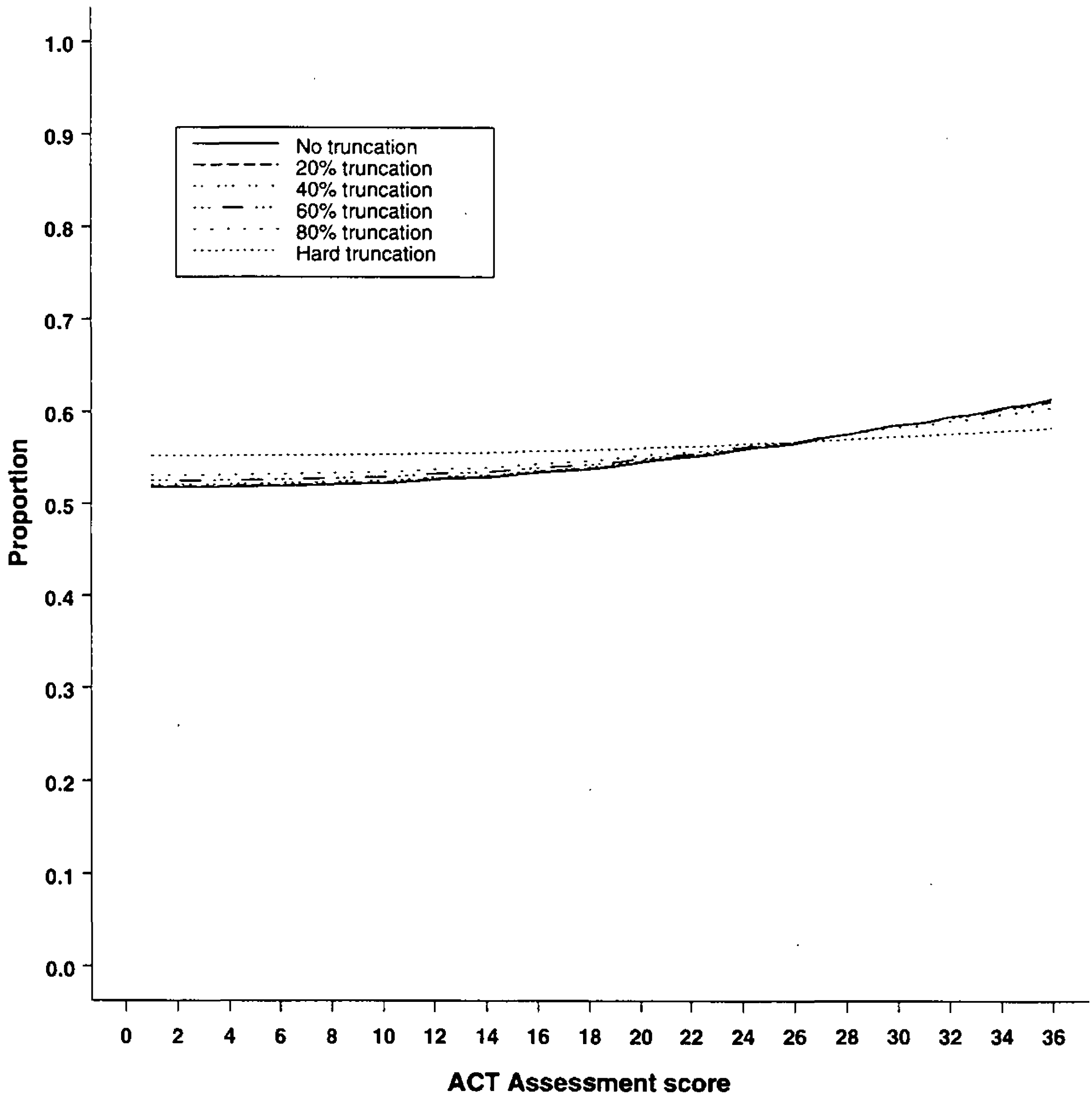


FIGURE B.5. Effects of Soft Truncation on Estimated Success Rate

(Placement Group 5: Flat slope, medium skewness)

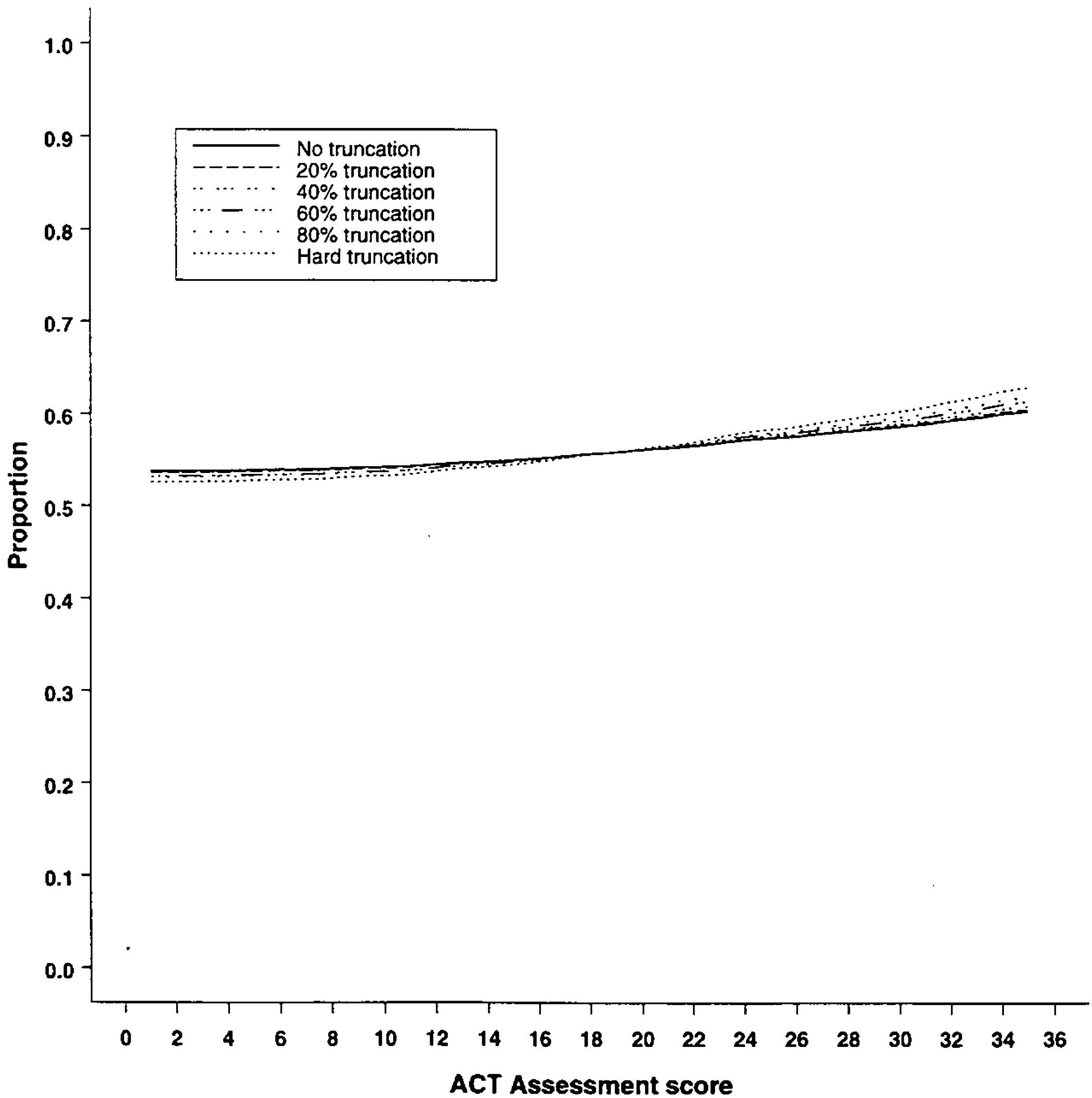


FIGURE B.6. Effects of Soft Truncation on Estimated Success Rate

(Placement Group 6: Flat slope, zero skewness)