

ACT Research Report Series 2005-3

Using ACT Assessment® Scores to Set Benchmarks for College Readiness

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Abstract

In this report, we establish benchmarks of readiness for four common first-year college courses: English Composition, College Algebra, Social Science, and Biology. Using grade data from a large sample of colleges, we modeled the probability of success in these courses as a function of ACT test scores. Success was defined as a course grade of B or higher, and for each college the ACT test score that yields a .50 probability of success was identified. The median of these scores represents the college readiness benchmark—the score that would typically indicate a student is ready for first-year college courses. Benchmarks were obtained for four common first-year courses: English Composition, using the ACT English score as the predictor; College Algebra, using the ACT Mathematics score as the predictor; Social Science, using the ACT Reading score as the predictor; and Biology, using the ACT Science score as the predictor.

Using ACT Assessment Scores to Set Benchmarks for College Readiness

One of the biggest transitions in a student's career is from high school to college. College is currently viewed as the primary path to greater opportunity, to higher-paying and more rewarding jobs. As such, students, parents, and counselors put great emphasis on going to college. Of course, enrolling in college does not ensure success; a student must persist and obtain a college degree. Students who drop out usually do so in the first year of college (Choy, 2002), and one of the primary reasons is academic difficulty.

The ACT Assessment® provides an objective measure of students' academic achievement and readiness for college and includes four curriculum-based tests of educational development: English, mathematics, reading, and science. The ACT tests are designed to measure academic skills that are taught in typical college-preparatory curricula in high school and are necessary in the first year of college. High scores on these tests show that a student is proficient in these subject areas and is ready for college-level work. Thus, ACT scores may be used to help determine if a student is academically prepared for the first year of college.

Factors other than academic preparedness—such as motivation and good study habits—are also important to success in college (Robbins et. al, 2004). In addition, other measures—e.g., high school grade point average and the level of courses taken in high school—can offer significant information on academic preparedness. The advantage of using ACT Assessment scores is that they are standardized measures that sustain meaning across schools and years. The meaning of high school grade point average, on the other hand, can differ across high schools. In this study, we focus on ACT Assessment scores and their relationship to success in the first year of college.

The relationship between ACT Assessment scores and success in the first year of college has been well documented in previous studies (e.g., Noble & Sawyer, 2004; ACT, 1997). These studies use statistical methods, including linear regression and logistic regression, to relate ACT test scores to success in college. Other measures of success have been considered, including first-year college grade point average and grades in particular courses. These studies have shown positive relationships between ACT Assessment scores and success criteria, even after the effects of other factors (e.g., high school grades) have been statistically controlled for. In other words, the higher a student's ACT score, the more likely that the student possesses the skills and knowledge necessary for success in college.

In this study, grades in typical first-year college courses were modeled as a function of an ACT test score. The four courses studied were English Composition, College Algebra, Social Science, and Biology—courses that are taken by a large percentage of first-year students. The ACT Assessment scores used were English (with English Composition), Mathematics (with College Algebra), Reading (with Social Science), and Science (with Biology).

Many factors are associated with success in a particular course, just one of which is academic preparation. And, even at the same college courses may vary in content and grading practices. Thus, for any test, theoretically there is no score below which a student cannot possibly succeed or above which a student is certain to succeed. Therefore, in this study we modeled the probability of success in a course. If the test is a valuable tool for predicting success in a course, there should be an increasing probability of success as the test score increases.

For each course within each college studied, a cutoff score was chosen such that the probability of success was sufficiently high. For this study, we chose cutoff scores for which the probability of a grade of B or higher in the course is .50. The specific reasons for defining cutoff

score in this way are given in the Research Methods section of this report. For each of the four courses studied, the cutoff scores varied across colleges. To establish a “benchmark” cutoff score, we calculated the median of the cutoff scores across colleges. The results are summarized in Table 1 below.

TABLE 1
Benchmarks for College Readiness, by Subject Area

Course	ACT Assessment Test	Benchmark for Success
English Composition	English	18
College Algebra	Mathematics	22
Social Science	Reading	21
Biology	Science	24

The benchmark values represent a summary across many colleges and many students. Because the material covered in a course and the grading practices within the course vary among colleges, these scores are not necessarily appropriate for every college. Instead, the *benchmark values* represent predictive indicators of success for *typical* students at *typical* colleges. They give students, parents, and counselors an easy and reliable guide—a standardized point of reference—as to whether a student has the knowledge and skills needed to have a reasonable chance of success in college.

Research Data

The data for this study came from colleges that had participated in ACT’s Course Placement Service, in which colleges send ACT student grades from a variety of courses. To obtain ACT test scores for each student, the student data were matched to the ACT Assessment data file using Social Security Number as the matching field. For each course, all colleges that supplied data for that course were included. If a college sent data from more than a single year,

only data from the most recent year were included. In order to increase our sample size for the Social Science analysis, we considered five different courses (history, psychology, sociology, political science, and economics) because each is typically reading intensive. The number and types of colleges varied for the four courses we studied. Table 2 summarizes the colleges included in each sample by college type (two-year or four-year), region of the United States, enrollment size, and the number of students included in the sample. As Table 2 shows, the sample includes both two- and four-year colleges from each region and colleges of different enrollment sizes.

For English Composition, the sample contained 46 two-year and 46 four-year institutions. Since two-year and four-year colleges are equally likely to participate in the ACT Course Placement Service, it is not surprising that our sample has an equal number of each. Given that most ACT-tested students enroll at four-year colleges, it may seem misleading to apply study findings to ACT-tested students when the study itself is based heavily on two-year colleges. However, as detailed in Research Methods section (see page 13), we weighted the sample with respect to the academic preparation of the college's students in order to make the sample representative of all colleges. By so doing, we also addressed the oversampling of two-year colleges.

As shown in Table 2, the colleges in the four samples are not evenly dispersed with respect to geographical distribution. This is not problematic because we do not expect geography to influence the relationship between ACT test scores and success in first-year college courses. One reason for the geographical imbalance is that the ACT Assessment is more widely used in some areas than others. For instance, it is not widely used in the western region of the country;

thus, we expect less representation from this region in our sample than from a random sample of colleges.

TABLE 2

Description of Colleges in Sample

Course	Type	Region	Enrollment	Sample Size
English Composition	46 two-year 46 four-year	49% Southwest 27% Midwest 17% East 7% West	1 st quartile: 1,137 Median: 2,475 3 rd quartile: 7,014	Median: 322 Mean: 827
College Algebra	40 two-year, 45 four-year	46% Southwest 24% Midwest 20% East 11% West	1 st quartile: 1,398 Median: 2,699 3 rd quartile: 6,916	Median: 130 Mean: 398
Social Sciences	21 two-year, 24 four-year	27% Southwest 22% Midwest 38% East 13% West	1 st quartile: 1,700 Median: 4,119 3 rd quartile: 11,488	Median: 413 Mean: 1,193
Biology	17 two-year, 14 four-year	13% Southwest 29% Midwest 39% East 19% West	1 st quartile: 1,746 Median: 3,341 3 rd quartile: 11,836	Median: 76 Mean: 462

TABLE 3

Description of Students in Sample

Course	Sample Size	ACT Scores	% Successful
English Composition	76,122	1 st quartile: 18 Median: 21 3 rd quartile: 24	61%
College Algebra	33,803	1 st quartile: 18 Median: 20 3 rd quartile: 23	45%
Social Science	53,705	1 st quartile: 18 Median: 22 3 rd quartile: 26	54%
Biology	14,136	1 st quartile: 20 Median: 23 3 rd quartile: 25	45%

The median enrollment of full-time students at colleges in the English Composition sample was 2,475. The typical institutions represented in the College Algebra, Social Science, and Biology samples were larger (median enrollments of 2,699, 4,119, and 3,341, respectively). For English Composition, the median size of the sample within a college was 322, with a mean of 827. (“Size of sample” refers to the number of students for whom we have an English Composition grade and an ACT English test score.) The fact that the mean sample size was so much larger than the median sample size indicates that some colleges in our sample provided considerably more data than the “average” college in our sample. Relative to English Composition, colleges in the College Algebra and Biology samples typically had fewer students with course grade data. Partly due to the fact that multiple courses were considered, colleges in the Social Science sample typically had more students with course grade data.

For each college in each sample, we considered the average ACT Composite score of all enrollees within the past two years as an indicator of the academic performance of the college’s students. The median of the average ACT Composite scores was 19 for English Composition, 19.1 for College Algebra, 19.3 for Social Science, and 19.2 for Biology.

In addition to characteristics of the colleges, we also considered characteristics of the students sampled (see Table 3). For English Composition, the total student sample size across all colleges was 76,122 and 61% of these students had a grade of B or higher in English Composition. The median ACT English score was 21, with a first quartile value of 18 and a third quartile value of 24. This can be compared to the ACT-tested graduating classes of 2002 through 2004. For this group, the median ACT English score was 20, with a first quartile value of 16 and a third quartile value of 24. We expected the sample to score higher than recent ACT-tested graduating classes since some students who take the ACT Assessment do not go to college.

For College Algebra, Social Science, and Biology, the total student sample sizes were considerably smaller than that of English Composition (33,803, 53,705, and 14,136, respectively). For College Algebra, the median ACT Mathematics score was 20, compared to the median score of 19 for recent ACT-tested graduating classes. For Social Science, the median ACT Reading score was 22, compared to the median score of 21 for recent ACT-tested graduates. For Biology, the median ACT Science score was 23, compared to the median score of 21 for recent ACT-tested graduates. Relative to the English Composition sample, the overall success rates were considerably lower for College Algebra and Biology: only 45% of the students had a grade of B or higher in both College Algebra and Biology. The overall success rate was also lower for Social Science as 54% were successful.

Research Methods

The Model

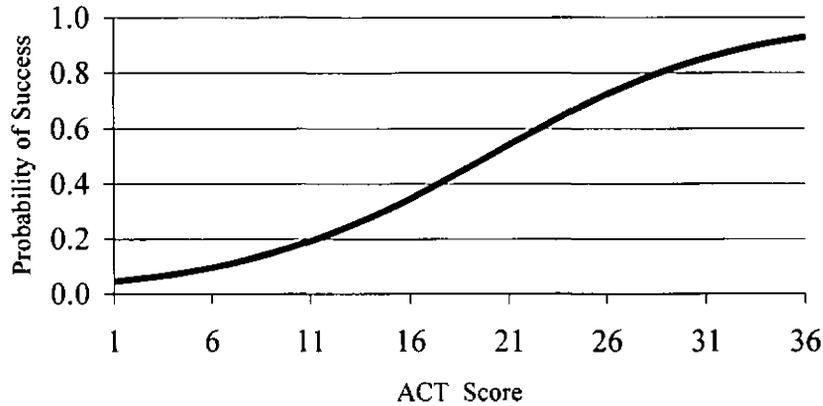
We investigated the relationship between an ACT test score and the grade a student achieves in a course by using a logistic regression model. In this model, course grades are divided into two categories: “success” and “failure.” The probability of a success was then modeled as a function of the test score. This model is more appropriate for typical uses of test scores (e.g., admissions and course placement) than the more common linear regression/correlation model (see Sawyer, 1989, for further details).

Using the logistic regression model, we modeled the probability of success as a particular function of test score given by

$$\text{Probability of Success} = \frac{\exp(\alpha + \beta x)}{1 + \exp(\alpha + \beta x)},$$

where x represents a test score, exp represents the exponential function, and α and β represent an intercept and a slope that vary across colleges. To illustrate a logistic regression model, Figure 1 shows a graph of a typical relationship between ACT test score and probability of success.

FIGURE 1. Example of a Logistic Regression Model for Probability of Success by Test Score



Note that the curve increases as test score increases, indicating that students with higher scores have a greater probability of success in the course. The probability of success starts at a fairly low value and climbs to almost 1.0 when the test score is very high. The logistic model can accommodate a wide variety of shapes of this curve, depending on the values of α and β . The value of α determines the starting point of the curve, while β determines the slope of the curve.

The values of α and β were estimated for each college in our study sample and reflect the relationship between proportions of successful students and individual test scores. For example, a particular college might have had a 20% success rate for students with a test score of 15, a 50% success rate for students with a test score of 20, and an 80% success rate for students with a test score of 25. The values of α and β were chosen so that the curve for the college matched the observed proportions as closely as possible.

The curves for each college were estimated using a hierarchical logistic regression model. In this type of model, the curves are assumed to have some similarity to one another. For colleges with small sample sizes, estimation of α and β is more precise because we utilize the college's similarity to other colleges with larger sample sizes. As an alternative to hierarchical logistic regression, we could allow the estimation of α and β to be determined solely by the grades and test scores from each college. The drawback to this approach is that because some colleges provide only a small number of grades and test scores for a particular course, the estimates of α and β are insufficiently precise. The hierarchical model allows for the inclusion of colleges with small samples while at the same time improving the precision of the estimates of α and β by utilizing the college's similarity to the other colleges. (For more on the advantages of hierarchical regression models, see Raudenbush and Bryk, 2002.) For more details on the statistical procedures we used, see the Appendix of this report.

Choice of the Success Criterion

To model the probability of success in the courses, we first determined what it means to be "successful" in a college course. Many would argue that a grade of C or higher is the correct definition, since a student who obtains such a grade has passed the course. However, in this study we defined success as a grade of B or higher in a course and identified three reasons why this option is preferable to the criterion of C or higher.

The first reason involves actual grading practices at colleges. Today, it's common for more than 50% of students in a course to earn grades of A or B. If course placement decisions were based on the student having at least .50 probability of earning a B or higher, then more than 50% of students would earn a B or higher (50% of the minimally qualified students would earn a B or higher), which is consistent with actual grading practices. On the other hand, if course

placement decisions were made based on the student having at least a .50 probability of a C or higher, then a smaller proportion of students would earn a B or higher, which would not be consistent with actual grading practices. Thus, we believe that the score identified as necessary for a .50 probability of earning a B or higher is most consistent with actual grading practices used by colleges.

Another argument for using a B or higher success criterion is related to the policy implications of course placement. The score that gives the most accurate predictions is the score that would give a probability of success of .50. That is, any student who has a greater than .50 probability would be predicted to be successful, and any student with less than a .50 probability of success would be predicted to be unsuccessful. The result is that the least qualified student in the class would have about a 50% chance of being unsuccessful. If success is defined as a grade of C or higher, that means that the least qualified student would have about a 50% chance of earning a grade of D or F in the course. It would seem poor policy to place a student into a course when there's a 50% chance of earning poor grades. This is not the case with the B or higher criterion, as the least qualified student has a reasonable probability of earning at least a C.

Finally, due in part to "grade inflation," grades below C are fairly uncommon in most courses (Johnson, 2003). The statistical model we used attempts to predict the probability of success in a course, and the estimated cutoff scores will be less precise if either a success or a failure happens only rarely. Also, the parameter estimates that define the relationship between the probability of success and test score can be badly affected by a single unusual observation when there are few failures. To protect against this problem, the B or higher criterion is a superior choice.

For these three reasons, we believe the B or higher criterion is the best choice for the definition of success. This definition has been successfully used in ACT's Course Placement Service in the past and yields better results than those obtained with the C or higher criterion. Although the college readiness benchmarks were determined by the B or higher criterion, we also discuss results related to the C or higher criterion in the Results section of this report.

For each college and each course, the values of α and β were estimated using the hierarchical logistic regression model. From each estimated curve, the first point at which the probability of success exceeded 0.5 was selected. This is referred to as the college's cutoff score for the course. From a decision theory point of view, this score point most accurately classifies the group into those who would be successful and those who would not be successful (see Sawyer, 1989).

Sample Weighting

It might be argued that since colleges in this study chose to participate in the Course Placement Service, they may not be representative of all colleges. In particular, there might be concern that the characteristics of the colleges in the sample might make their cutoff scores different from those of colleges that were not in the study. As previously mentioned, it is implausible that region of the country affects the relationship between test scores and grades in first-year college courses. Likewise, it seems improbable that a college's enrollment size or affiliation (public vs. private) would directly affect the relationship. However, one might argue that colleges whose students have different levels of academic preparation might have courses of different levels of difficulty and with different standards of grading. Therefore, we wanted our sample of colleges to be representative of all colleges in terms of students' academic preparation.

To make our sample more representative with respect to students' academic preparation, we weighted the individual cutoff scores. That is, to achieve a more representative distribution with respect to academic preparation, we counted some colleges more than others. The value that we used to measure the academic preparation of a college's students was the average ACT Composite score. A higher average ACT Composite score indicates that students are better prepared academically. For each college, the average ACT Composite score was calculated from the scores of all enrollees from the previous two years.

For each sample, we grouped the colleges by their average ACT Composite score. Since multiple courses with colleges were considered for the Social Science sample, we grouped each college/course combination by the average ACT Composite score of the college. The percentage of colleges (or college and course combinations) in each group for each of the four samples was compared to the percentage of colleges in that group nationwide. Each college (or college and course combinations) was then given a weight that was the ratio of the population percentage to the sample percentage. The definition of the groups and the weights for each group is given in Table 4 for English Composition, Table 5 for College Algebra, Table 6 for Social Science, and Table 7 for Biology.

Using Table 7 as a guide, we can see that the sample proportion of colleges with an average ACT Composite score less than or equal to 17.62 was .23, while for the general population, the proportion was only .16. Thus, in our study, we over-sampled colleges in this range of mean ACT Composite score. If the cutoff scores vary by range, this could affect our results. Thus, each college was given a weight of .69, which had the effect of counting these colleges less than others in our sample. Note that some colleges had weights greater than one and others had weights less than one. This depended solely on whether the colleges in that group of

mean ACT Composite scores were over- or under-sampled. If all of the weights had been one, then our sample would have been perfectly representative of the population with respect to mean ACT Composite score. The purpose of weighting was to make the sample more representative of the population with respect to students' academic preparation.

In general, we tried to make the groups of about equal size and resulting in weights that were not drastically greater or less than one. With extremely large weights, we would be concerned that colleges in that group would be counted too heavily and unduly influence the sample median. Likewise, with extremely small weights, we would be concerned that colleges in that group would contribute little to the sample median.

TABLE 4

Weights for the English Composition Course Sample

Mean ACT Composite	Population Proportion	Sample		Weight
		N	Proportion	
≤ 17.62	0.16	21	0.23	0.68
17.63-18.62	0.13	20	0.22	0.61
18.63-20.37	0.27	26	0.28	0.96
20.38-21.62	0.15	10	0.11	1.35
≥ 21.63	0.29	15	0.16	1.80
Total	1.00	93	1.00	

TABLE 5

Weights for the College Algebra Course Sample

Mean ACT Composite	Population Proportion	Sample		Weight
		N	Proportion	
≤ 17.37	0.13	10	0.12	1.12
17.38-19.37	0.28	37	0.44	0.63
19.38-21.12	0.26	18	0.21	1.21
21.13-21.87	0.06	10	0.12	0.50
≥ 21.88	0.28	10	0.12	2.36
Total	1.00	85	1.00	

TABLE 6

Weights for the Social Science Course Sample

Mean ACT Composite	Population Proportion	Sample		Weight
		N	Proportion	
≤ 17.62	0.16	19	0.19	0.85
17.63-18.62	0.13	14	0.14	0.97
18.63-20.37	0.27	28	0.27	0.99
20.38-21.62	0.15	12	0.12	1.24
≥ 21.63	0.29	29	0.28	1.02
Total	1.00	102	1.00	

TABLE 7

Weights for the Biology Course Sample

Mean ACT Composite	Population Proportion	Sample		Weight
		N	Proportion	
≤ 17.62	0.16	7	0.23	0.69
17.63-18.62	0.13	4	0.13	1.02
18.63-20.37	0.27	8	0.26	1.06
20.38-21.62	0.15	4	0.13	1.14
≥ 21.63	0.29	8	0.26	1.13
Total	1.00	31	1.00	

Results

Recall that the cutoff scores may vary from college to college. To describe the variability in cutoff scores across colleges, three values were found: the median, the first quartile, and the third quartile. These values are given in Table 8 below.

TABLE 8

Summary of the Frequency Distribution of the Cutoff Scores

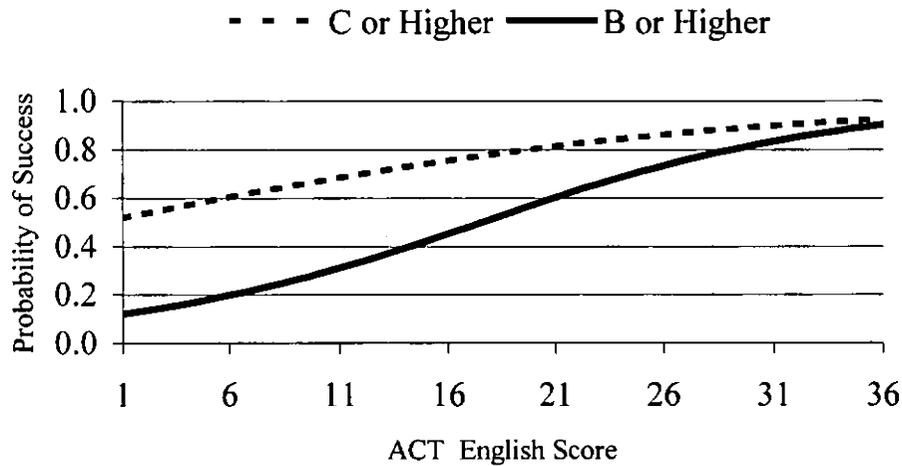
Course and Test	1 st quartile	Median	3 rd quartile
English Composition and ACT English Score	14	18	20
College Algebra and ACT Mathematics Score	22	22	23
Social Science and ACT Reading Score	17	21	24
Biology and ACT Science Score	22	24	25

The median values were considered typical values and so represent a good summary of the distribution of cutoff scores. Thus, the scores of 18 for English, 22 for College Algebra, 21 for Social Science, and 24 for Biology represent benchmark ACT test scores that would give a student at the typical college a reasonable chance of success in these courses. Students with scores below the benchmark values would not be considered ready for standard college courses and may be in need of developmental work. Since the medians of the cutoff scores represented typical values needed for a .50 probability of success, we can use these values as overall standards for college readiness.

Although they are useful predictors of success in first-year college courses, ACT scores above the cutoffs do not guarantee success. For English Composition, 73% of the students in the sample had an ACT English score greater than, or equal to, their college's cutoff score. Of these students, 69% earned a grade of B or higher, while 86% earned a grade of C or higher. In comparison, 39% of the students below their college's cutoff earned a grade of B or higher, while 72% earned a grade of C or higher.

In Figure 2, we present the relationship between ACT English score and probability of success in English Composition for the typical college in our sample. Two probability curves are given: one represents the probability of a B or higher, and the other represents the probability of a C or higher. From Figure 2, we see that a student with a benchmark ACT English score of 18 has an 80% chance of earning a C or higher in English Composition at the typical college.

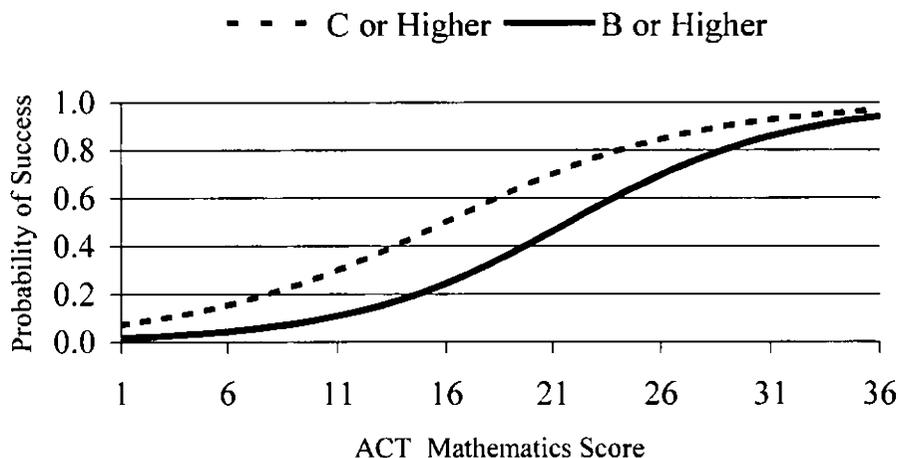
FIGURE 2. ACT English Score and Probability of Success in English Composition



For College Algebra, 36% of the students in the sample had an ACT Mathematics score greater than, or equal to, their college's cutoff score. Of these students, 64% earned a grade of B or higher, while 81% earned a grade of C or higher. In comparison, 34% of the students below their college's cutoff earned a grade of B or higher, while 61% earned a grade of C or higher.

In Figure 3, we present the relationship between ACT Mathematics score and probability of success (B or higher and C or higher) in College Algebra for the typical college in our sample. From Figure 3, we see that a student with a benchmark ACT Mathematics score of 22 has a 75% chance of earning a C or higher in College Algebra at the typical college.

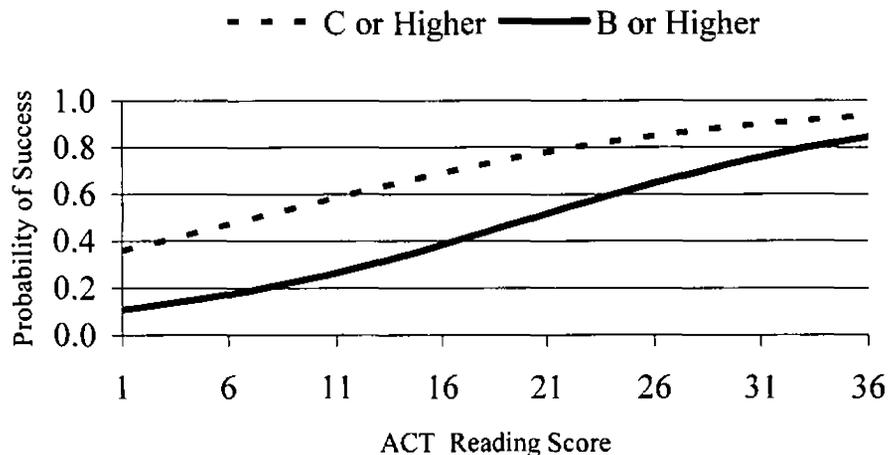
FIGURE 3. ACT Mathematics Score and Probability of Success in College Algebra



For Social Science, 62% of the students in the sample had an ACT Reading score greater than, or equal to, their college's cutoff score. Of these students, 64% earned a grade of B or higher, while 86% earned a grade of C or higher. In comparison, 37% of the students below their college's cutoff earned a grade of B or higher, while 68% earned a grade of C or higher.

In Figure 4, we present the relationship between ACT Reading score and probability of success (B or higher and C or higher) in Social Science for the typical course in our sample. From Figure 4, we see that a student with a benchmark ACT Reading score of 21 has a 78% chance of earning a C or higher in Social Science at the typical college.

FIGURE 4. ACT Reading Score and Probability of Success in Social Science

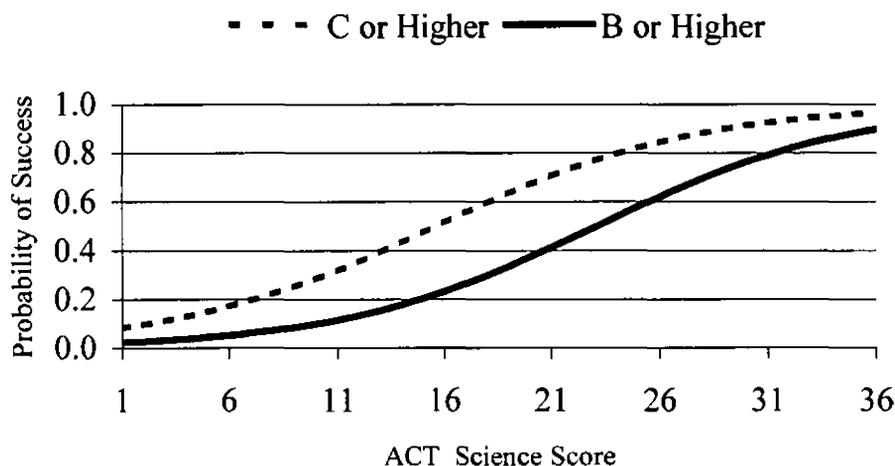


For Biology, 36% of the students in the sample had an ACT Science score greater than, or equal to, their college's cutoff score. Of these students, 62% earned a grade of B or higher, while 86% earned a grade of C or higher. In comparison, 36% of the students below their college's cutoff earned a grade of B or higher, while 68% earned a grade of C or higher.

In Figure 5, we present the relationship between ACT Science score and probability of success (B or higher and C or higher) in Biology for the typical college in our sample. From

Figure 5, we see that a student with a benchmark ACT Science score of 24 has an 80% chance of earning a C or higher in Biology at the typical college.

FIGURE 5. ACT Science Score and Probability of Success in Biology



Conclusion

Before investing the substantial resources needed to go to college, students and their parents need to know what it takes to be successful in college. In particular, students need to know if they are adequately prepared academically for the courses they will take. To help answer that question, we analyzed ACT test scores and grades in standard first-year courses for a representative group of colleges and established scores on the appropriate ACT tests that represent a .50 probability of earning a B or higher in each course at a typical college. We think of these scores as benchmarks for success in these courses.

The courses considered were English Composition, College Algebra, Social Science, and Biology. For the English Composition course, the test score representing the benchmark for success was an 18 on the ACT English test. For College Algebra, the benchmark score was a 22 on the ACT Mathematics test. For Social Science, the benchmark score was a 21 on the ACT Reading test. For Biology, the benchmark score was a 24 on the ACT Science test. Students,

parents, and counselors can use these scores to determine the academic areas in which students are ready for college coursework and the areas in which they need more work.

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Appendix: Hierarchical Logistic Regression Model

For each college/course combination included in the sample, a cutoff score was found that represents the ACT test score that gives a .50 probability of success in the college/course. We modeled the probability of success using the logistic function as follows:

$$\text{Probability of Success} = \frac{\exp(\alpha + \beta x)}{1 + \exp(\alpha + \beta x)},$$

where x represents a test score, \exp represents the exponential function, and α and β represent an intercept and slope that vary across college/course combinations. The test score that gives a .50 probability of success is found by solving the probability equation for x as follows:

$$.50 = \frac{\exp(\alpha + \beta x)}{1 + \exp(\alpha + \beta x)} \text{ implies that } \exp(\alpha + \beta x) = 1 \text{ and } \alpha + \beta x = 0.$$

Hence, $x = -\alpha / \beta$ is the test score that gives a .50 probability of success. Since scores on the ACT test can only take integer values, the cutoff score was set to $-\alpha / \beta$, rounded to the next highest integer.

To estimate the values of α and β for each college/course combination, a random-coefficients logistic regression model was used. This type of model is often referred to as a hierarchical logistic regression model. For each course, we assumed that a college's α and β coefficients are drawn from a bivariate-normal distribution, with mean vector denoted (μ_α, μ_β) , and an unstructured variance-covariance matrix denoted Σ . For each course, the parameter estimates, standard errors, and 95% confidence intervals are given in Table 9. The implications of this model are that, for each college, α and β are estimated based upon data for that college, as well as the average values of α and β across colleges. The NLMIXED procedure within SAS

statistical software was used to fit the model and obtain the estimates of α and β for each college.

For more details on SAS PROC NLMIXED, see SAS (1999).

TABLE 9

Parameter Estimates From Hierarchical Logistic Regression Models

English Composition and ACT English Score				
Parameter	Estimate	Standard Error	95% confidence interval	
			Lower Bound	Upper Bound
μ_α	-2.1195	.1389	-2.3954	-1.8437
μ_β	.1204	.0055	.1095	.1313
Σ_{11}	1.2849	.2509	.7864	1.7835
Σ_{12}	-.04286	.0093	-.0613	-.02442
Σ_{22}	.00167	.00036	.00095	.00239
College Algebra and ACT Mathematics Score				
Parameter	Estimate	Standard Error	95% confidence interval	
			Lower Bound	Upper Bound
μ_α	-4.2524	.2086	-4.6672	-3.8375
μ_β	.1954	.0094	.1767	.214
Σ_{11}	2.0689	.5433	.9884	3.1494
Σ_{12}	-.0819	.0227	-.1270	-.03686
Σ_{22}	.00383	.00099	.0019	.0058
Social Science and ACT Reading Score				
Parameter	Estimate	Standard Error	95% confidence interval	
			Lower Bound	Upper Bound
μ_α	-2.2121	.1304	-2.4708	-1.9533
μ_β	.1085	.0046	.0994	.1177
Σ_{11}	1.2805	.2471	.7902	1.7709
Σ_{12}	-.0365	.0082	-.0527	-.0202
Σ_{22}	.00128	.00029	.00070	.00186

Biology and ACT Science Score				
Parameter	Estimate	Standard Error	95% confidence interval	
			Lower Bound	Upper Bound
μ_{α}	-3.8933	.2845	-4.4751	-3.3114
μ_{β}	.1687	.0120	.1441	.1933
Σ_{11}	.9104	.4358	.01903	1.8018
Σ_{12}	-.0254	.0145	-.0550	.0042
Σ_{22}	.0008	.0005	0	.0019

